

Two-Dimensional Motion and Vectors

Two-Dimensional Motion and Vectors, Section 1 Review

Givens

2. $\Delta x_1 = 85 \text{ m}$
 $d_2 = 45 \text{ m}$
 $\theta_2 = 30.0^\circ$

Solutions

Students should use graphical techniques. Their answers can be checked using the techniques presented in Section 2. Answers may vary.

$$\Delta x_2 = d_2(\cos \theta_2) = (45 \text{ m})(\cos 30.0^\circ) = 39 \text{ m}$$

$$\Delta y_2 = d_2(\sin \theta_2) = (45 \text{ m})(\sin 30.0^\circ) = 22 \text{ m}$$

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 85 \text{ m} + 39 \text{ m} = 124 \text{ m}$$

$$\Delta y_{tot} = \Delta y_2 = 22 \text{ m}$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(124 \text{ m})^2 + (22 \text{ m})^2}$$

$$d = \sqrt{15\,400 \text{ m}^2 + 480 \text{ m}^2} = \sqrt{15\,900 \text{ m}^2} = \boxed{126 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{22 \text{ m}}{124 \text{ m}}\right) = \boxed{(1.0 \times 10^1)^\circ \text{ above the horizontal}}$$

3. $v_{y,1} = 2.50 \times 10^2 \text{ km/h}$
 $v_2 = 75 \text{ km/h}$
 $\theta_2 = -45^\circ$

Students should use graphical techniques.

$$v_{x,2} = v_2(\cos \theta_2) = (75 \text{ km/h})[\cos(-45^\circ)] = 53 \text{ km/h}$$

$$v_{y,2} = v_2(\sin \theta_2) = (75 \text{ km/h})[\sin(-45^\circ)] = -53 \text{ km/h}$$

$$v_{y,tot} = v_{y,1} + v_{y,2} = 2.50 \times 10^2 \text{ km/h} - 53 \text{ km/h} = 197 \text{ km/h}$$

$$v_{x,tot} = v_{x,2} = 53 \text{ km/h}$$

$$v = \sqrt{(v_{x,tot})^2 + (v_{y,tot})^2} = \sqrt{(53 \text{ km/h})^2 + (197 \text{ km/h})^2}$$

$$v = \sqrt{2800 \text{ km}^2/\text{h}^2 + 38\,800 \text{ km}^2/\text{h}^2} = \sqrt{41\,600 \text{ km}^2/\text{h}^2} = \boxed{204 \text{ km/h}}$$

$$\theta = \tan^{-1}\left(\frac{v_{y,tot}}{v_{x,tot}}\right) = \tan^{-1}\left(\frac{204 \text{ km/h}}{53 \text{ km/h}}\right) = \boxed{75^\circ \text{ north of east}}$$

4. $v_{y,1} = \frac{2.50 \times 10^2 \text{ km/h}}{2}$
 $= 125 \text{ km/h}$
 $v_{x,2} = 53 \text{ km/h}$
 $v_{y,2} = -53 \text{ km/h}$

Students should use graphical techniques.

$$v_{y,dr} = v_{y,1} + v_{y,2} = 125 \text{ km/h} - 53 \text{ km/h} = 72 \text{ km/h}$$

$$v_{x,dr} = v_{x,2} = 53 \text{ km/h}$$

$$v = \sqrt{(v_{x,dr})^2 + (v_{y,dr})^2} = \sqrt{(53 \text{ km/h})^2 + (72 \text{ km/h})^2}$$

$$v = \sqrt{2800 \text{ km}^2/\text{h}^2 + 5200 \text{ km}^2/\text{h}^2} = \sqrt{8.0 \times 10^3 \text{ km}^2/\text{h}^2} = \boxed{89 \text{ km/h}}$$

$$\theta = \tan^{-1}\left(\frac{v_{y,dr}}{v_{x,dr}}\right) = \tan^{-1}\left(\frac{72 \text{ km/h}}{53 \text{ km/h}}\right) = \boxed{54^\circ \text{ north of east}}$$

Two-Dimensional Motion and Vectors, Practice A

Givens

- $\Delta \mathbf{x}_1 = 8 \text{ km east}$
 $\Delta x_1 = 8 \text{ km}$
 $\Delta \mathbf{x}_2 = 3 \text{ km west} = -3 \text{ km, east}$
 $\Delta x_2 = 3 \text{ km}$
 $\Delta \mathbf{x}_3 = 12 \text{ km east}$
 $\Delta x_3 = 12 \text{ km}$
 $\Delta y = 0 \text{ km}$

Solutions

- $d = \Delta x_1 + \Delta x_2 + \Delta x_3 = 8 \text{ km} + 3 \text{ km} + 12 \text{ km} = \boxed{23 \text{ km}}$
- $\Delta \mathbf{x}_{\text{tot}} = \Delta \mathbf{x}_1 + \Delta \mathbf{x}_2 + \Delta \mathbf{x}_3 = 8 \text{ km} + (-3 \text{ km}) + 12 \text{ km} = \boxed{17 \text{ km east}}$

- $\Delta x = 7.5 \text{ m}$
 $\Delta y = 45.0 \text{ m}$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(7.5 \text{ m})^2 + (45.0 \text{ m})^2}$$
$$d = \sqrt{56 \text{ m}^2 + 2020 \text{ m}^2} = \sqrt{2080 \text{ m}^2} = \boxed{45.6 \text{ m}}$$

Measuring direction with respect to y (north),

$$\theta = \tan^{-1}\left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{7.5 \text{ m}}{45.0 \text{ m}}\right) = \boxed{9.5^\circ \text{ east of due north}}$$

- $\Delta x = 6.0 \text{ m}$
 $\Delta y = 14.5 \text{ m}$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(6.0 \text{ m})^2 + (14.5 \text{ m})^2}$$
$$d = \sqrt{36 \text{ m}^2 + 2.10 \times 10^2 \text{ m}^2} = \sqrt{246 \text{ m}^2} = \boxed{15.7 \text{ m}}$$

Measuring direction with respect to the length of the field (down the field),

$$\theta = \tan^{-1}\left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{6.0 \text{ m}}{14.5 \text{ m}}\right) = \boxed{22^\circ \text{ to the side of downfield}}$$

- $\Delta x = 1.2 \text{ m}$
 $\Delta y = -1.4 \text{ m}$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(1.2 \text{ m})^2 + (-1.4 \text{ m})^2}$$
$$d = \sqrt{1.4 \text{ m}^2 + 2.0 \text{ m}^2} = \sqrt{3.4 \text{ m}^2} = \boxed{1.8 \text{ m}}$$
$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{-1.4 \text{ m}}{1.2 \text{ m}}\right) = -49^\circ = \boxed{49^\circ \text{ below the horizontal}}$$

Two-Dimensional Motion and Vectors, Practice B

- $v = 105 \text{ km/h}$
 $\theta = 25^\circ$

$$v_x = v(\cos \theta) = (105 \text{ km/h})(\cos 25^\circ) = \boxed{95 \text{ km/h}}$$

- $v = 105 \text{ km/h}$
 $\theta = 25^\circ$

$$v_y = v(\sin \theta) = (105 \text{ km/h})(\sin 25^\circ) = \boxed{44 \text{ km/h}}$$

- $v = 22 \text{ m/s}$
 $\theta = 15^\circ$

$$v_x = v(\cos \theta) = (22 \text{ m/s})(\cos 15^\circ) = \boxed{21 \text{ m/s}}$$

$$v_y = v(\sin \theta) = (22 \text{ m/s})(\sin 15^\circ) = \boxed{5.7 \text{ m/s}}$$

- $d = 5 \text{ m}$
 $\theta = 90^\circ$

$$\Delta x = d(\cos \theta) = (5 \text{ m})(\cos 90^\circ) = \boxed{0 \text{ m}}$$

$$\Delta y = d(\sin \theta) = (5 \text{ m})(\sin 90^\circ) = \boxed{5 \text{ m}}$$

Two-Dimensional Motion and Vectors, Practice C

Givens

Solutions

1. $d_1 = 35 \text{ m}$
 $\theta_1 = 0.0^\circ$
 $d_2 = 15 \text{ m}$
 $\theta_2 = 25^\circ$

$$\Delta x_1 = d_1(\cos \theta_1) = (35 \text{ m})(\cos 0.0^\circ) = 35 \text{ m}$$

$$\Delta y_1 = d_1(\sin \theta_1) = (35 \text{ m})(\sin 0.0^\circ) = 0.0 \text{ m}$$

$$\Delta x_2 = d_2(\cos \theta_2) = (15 \text{ m})(\cos 25^\circ) = 14 \text{ m}$$

$$\Delta y_2 = d_2(\sin \theta_2) = (15 \text{ m})(\sin 25^\circ) = 6.3 \text{ m}$$

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 35 \text{ m} + 14 \text{ m} = 49 \text{ m}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 0.0 \text{ m} + 6.3 \text{ m} = 6.3 \text{ m}$$

$$d_{tot} = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(49 \text{ m})^2 + (6.3 \text{ m})^2}$$

$$d_{tot} = \sqrt{2400 \text{ m}^2 + 40 \text{ m}^2} = \sqrt{2440 \text{ m}^2} = \boxed{49 \text{ m}}$$

$$\theta_{tot} = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{6.3 \text{ m}}{49 \text{ m}}\right) = \boxed{7.3^\circ \text{ to the right of downfield}}$$

2. $d_1 = 2.5 \text{ km}$
 $\theta_1 = 35^\circ$
 $d_2 = 5.2 \text{ km}$
 $\theta_2 = 22^\circ$

$$\Delta x_1 = d_1(\cos \theta_1) = (2.5 \text{ km})(\cos 35^\circ) = 2.0 \text{ km}$$

$$\Delta y_1 = d_1(\sin \theta_1) = (2.5 \text{ km})(\sin 35^\circ) = 1.4 \text{ km}$$

$$\Delta x_2 = d_2(\cos \theta_2) = (5.2 \text{ km})(\cos 22^\circ) = 4.8 \text{ km}$$

$$\Delta y_2 = d_2(\sin \theta_2) = (5.2 \text{ km})(\sin 22^\circ) = 1.9 \text{ km}$$

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 2.0 \text{ km} + 4.8 \text{ km} = 6.8 \text{ km}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 1.4 \text{ km} + 1.9 \text{ km} = 3.3 \text{ km}$$

$$d_{tot} = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(6.8 \text{ km})^2 + (3.3 \text{ km})^2}$$

$$d_{tot} = \sqrt{46 \text{ km}^2 + 11 \text{ km}^2} = \sqrt{57 \text{ km}^2} = \boxed{7.5 \text{ km}}$$

$$\theta_{tot} = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{3.3 \text{ km}}{6.8 \text{ km}}\right) = \boxed{26^\circ \text{ above the horizontal}}$$

3. $d_1 = 8.0 \text{ m}$
 $\theta_1 = 90.0^\circ$
 $d_2 = 3.5 \text{ m}$
 $\theta_2 = 55^\circ$
 $d_3 = 5.0 \text{ m}$
 $\theta_3 = 0.0^\circ$

Measuring direction with respect to $x = (\text{east})$,

$$\Delta x_1 = d_1(\cos \theta_1) = (8.0 \text{ m})(\cos 90.0^\circ) = 0.0 \text{ m}$$

$$\Delta y_1 = d_1(\sin \theta_1) = (8.0 \text{ m})(\sin 90.0^\circ) = 8.0 \text{ m}$$

$$\Delta x_2 = d_2(\cos \theta_2) = (3.5 \text{ m})(\cos 55^\circ) = 2.0 \text{ m}$$

$$\Delta y_2 = d_2(\sin \theta_2) = (3.5 \text{ m})(\sin 55^\circ) = 2.9 \text{ m}$$

$$\Delta x_3 = d_3(\cos \theta_3) = (5.0 \text{ m})(\cos 0.0^\circ) = 5.0 \text{ m}$$

$$\Delta y_3 = d_3(\sin \theta_3) = (5.0 \text{ m})(\sin 0.0^\circ) = 0.0 \text{ m}$$

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 + \Delta x_3 = 0.0 \text{ m} + 2.0 \text{ m} + 5.0 \text{ m} = 7.0 \text{ m}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 + \Delta y_3 = 8.0 \text{ m} + 2.9 \text{ m} + 0.0 \text{ m} = 10.9 \text{ m}$$

$$d_{tot} = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(7.0 \text{ m})^2 + (10.9 \text{ m})^2}$$

$$d_{tot} = \sqrt{49 \text{ m}^2 + 119 \text{ m}^2} = \sqrt{168 \text{ m}^2} = \boxed{13.0 \text{ m}}$$

$$\theta_{tot} = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{10.9 \text{ m}}{7.0 \text{ m}}\right) = \boxed{57^\circ \text{ north of east}}$$

Givens

4. $d_1 = 75 \text{ km}$
 $\theta_1 = -30.0^\circ$
 $d_2 = 155 \text{ km}$
 $\theta_2 = 60.0^\circ$

Solutions

Measuring direction with respect to y (north),

$$\Delta x_1 = d_1(\sin \theta_1) = (75 \text{ km})(\sin -30.0^\circ) = -38 \text{ km}$$

$$\Delta y_1 = d_1(\cos \theta_1) = (75 \text{ km})(\cos -30.0^\circ) = 65 \text{ km}$$

$$\Delta x_2 = d_2(\sin \theta_2) = (155 \text{ km})(\sin 60.0^\circ) = 134 \text{ km}$$

$$\Delta y_2 = d_2(\cos \theta_2) = (155 \text{ km})(\cos 60.0^\circ) = 77.5 \text{ km}$$

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = -38 \text{ km} + 134 \text{ km} = 96 \text{ km}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 65 \text{ km} + 77.5 \text{ km} = 142 \text{ km}$$

$$d_{tot} = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(96 \text{ km})^2 + (142 \text{ km})^2} = \sqrt{9200 \text{ km}^2 + 20200 \text{ km}^2}$$

$$d_{tot} = \sqrt{29400 \text{ km}^2} = \boxed{171 \text{ km}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta x_{tot}}{\Delta y_{tot}}\right) = \tan^{-1}\left(\frac{96 \text{ km}}{142 \text{ km}}\right) = \boxed{34^\circ \text{ east of north}}$$

Two-Dimensional Motion and Vectors, Section 2 Review

2. $v_x = 3.0 \text{ m/s}$
 $v_y = 5.0 \text{ m/s}$

a. $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(3.0 \text{ m/s})^2 + (5.0 \text{ m/s})^2}$

$$v = \sqrt{9.0 \text{ m}^2/\text{s}^2 + 25 \text{ m}^2/\text{s}^2} = \sqrt{34 \text{ m}^2/\text{s}^2} = \boxed{5.8 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{5.0 \text{ m/s}}{3.0 \text{ m/s}}\right) = \boxed{59^\circ \text{ downriver from its intended path}}$$

- $v_x = 1.0 \text{ m/s}$
 $v_y = 6.0 \text{ m/s}$

b. $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(1.0 \text{ m/s})^2 + (6.0 \text{ m/s})^2}$

$$v = \sqrt{1.0 \text{ m}^2/\text{s}^2 + 36 \text{ m}^2/\text{s}^2} = \sqrt{37 \text{ m}^2/\text{s}^2} = \boxed{6.1 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{v_x}{v_y}\right) = \tan^{-1}\left(\frac{1.0 \text{ m/s}}{6.0 \text{ m/s}}\right) = \boxed{9.5^\circ \text{ from the direction the wave is traveling}}$$

3. $d = 10.0 \text{ km}$
 $\theta = 45.0^\circ$

a. $\Delta x = d(\cos \theta) = (10.0 \text{ km})(\cos 45.0^\circ) = \boxed{7.07 \text{ km}}$

$$\Delta y = d(\sin \theta) = (10.0 \text{ km})(\sin 45.0^\circ) = \boxed{7.07 \text{ km}}$$

- $a = 2.0 \text{ m/s}^2$
 $\theta = 35^\circ$

b. $a_x = a(\cos \theta) = (2.0 \text{ m/s}^2)(\cos 35^\circ) = \boxed{1.6 \text{ m/s}^2}$

$$a_y = a(\sin \theta) = (2.0 \text{ m/s}^2)(\sin 35^\circ) = \boxed{1.1 \text{ m/s}^2}$$

Two-Dimensional Motion and Vectors, Practice D

1. $\Delta y = -0.70 \text{ m}$
 $\Delta x = 0.25 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \frac{\Delta x}{v_x}$$

$$v_x = \sqrt{\frac{a_y}{2\Delta y}} \Delta x = \sqrt{\frac{-9.81 \text{ m/s}^2}{(2)(-0.70 \text{ m})}} (0.25 \text{ m}) = \boxed{0.66 \text{ m/s}}$$

2. $\Delta y = -1.0 \text{ m}$
 $\Delta x = 2.2 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \frac{\Delta x}{v_x}$$

$$v_x = \sqrt{\frac{a_y}{2\Delta y}} \Delta x = \sqrt{\frac{-9.81 \text{ m/s}^2}{(2)(-1.0 \text{ m})}} (2.2 \text{ m}) = \boxed{4.9 \text{ m/s}}$$

Givens

3. $\Delta y = -5.4 \text{ m}$
 $\Delta x = 8.0 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

Solutions

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \frac{\Delta x}{v_x}$$

$$v_x = \sqrt{\frac{a_y}{2\Delta y}} \Delta x = \sqrt{\frac{-9.81 \text{ m/s}^2}{(2)(-5.4 \text{ m})}} (8.0 \text{ m}) = \boxed{7.6 \text{ m/s}}$$

4. $v_x = 7.6 \text{ m/s}$
 $\Delta y = -2.7 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \frac{\Delta x}{v_x}$$

$$\Delta x = \sqrt{\frac{2\Delta y}{a_y}} v_x = \sqrt{\frac{(2)(-2.7 \text{ m})}{-9.81 \text{ m/s}^2}} (7.6 \text{ m/s}) = \boxed{5.6 \text{ m}}$$

Two-Dimensional Motion and Vectors, Practice E

1. $\Delta x = 4.0 \text{ m}$
 $\theta = 15^\circ$
 $v_i = 5.0 \text{ m/s}$
 $\Delta y_{\text{max}} = -2.5 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta x = v_i(\cos \theta)\Delta t$$

$$\Delta t = \frac{\Delta x}{v_i(\cos \theta)} = \frac{4.0 \text{ m}}{(5.0 \text{ m/s})(\cos 15^\circ)} = 0.83 \text{ s}$$

$$\Delta y = v_i(\sin \theta)\Delta t + \frac{1}{2}a_y\Delta t^2 = (5.0 \text{ m/s})(\sin 15^\circ)(0.83 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(0.83 \text{ s})^2$$

$$\Delta y = 1.1 \text{ m} - 3.4 \text{ m} = \boxed{-2.3 \text{ m}} \quad \boxed{\text{yes}}$$

2. $\Delta x = 301.5 \text{ m}$
 $\theta = 25.0^\circ$

At Δy_{max} , $v_{yf} = 0 \text{ m/s}$, $\Delta t = \Delta t_{\text{peak}}$

$$v_{yf} = v_i \sin \theta + a_y \Delta t_{\text{peak}} = 0$$

$$\Delta t_{\text{peak}} = \frac{-v_i \sin \theta}{a_y}$$

at Δx_{max} , $\Delta t_m = 2 \Delta t_p = \frac{-2v_i \sin \theta}{a_y}$

$$\Delta x_{\text{max}} = v_i \cos \theta \Delta t_m = v_i \cos \theta \left(\frac{-2v_i \sin \theta}{a_y} \right) = \frac{-2v_i^2 \sin \theta \cos \theta}{a_y}$$

$$v_i = \sqrt{\frac{-a_y \Delta x_{\text{max}}}{2 \sin \theta \cos \theta}}$$

$$v_{fy}^2 = v_i^2 (\sin \theta)^2 = -2a_y \Delta y_{\text{max}} = 0 \text{ at peak}$$

$$\Delta y_{\text{max}} = \frac{v_i^2 (\sin \theta)^2}{-2a_y} = \left(\frac{-a_y \Delta x_{\text{max}}}{2 \sin \theta \cos \theta} \right) \left(\frac{(\sin \theta)^2}{-2a_y} \right) = \frac{1}{4} \Delta x_{\text{max}} \tan \theta$$

$$\Delta y_{\text{max}} = \frac{1}{4} (301.5 \text{ m}) (\tan 25.0^\circ) = \boxed{35.1 \text{ m}}$$

3. $\Delta x = 42.0 \text{ m}$
 $\theta = 25^\circ$
 $v_i = 23.0 \text{ m/s}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta t = \frac{\Delta x}{v_x} = \frac{\Delta x}{v_i (\cos \theta)} = \frac{42.0 \text{ m}}{(23.0 \text{ m/s})(\cos 25^\circ)} = \boxed{2.0 \text{ s}}$$

At maximum height, $v_{yf} = 0 \text{ m/s}$.

$$v_{yf}^2 = v_{yi}^2 + 2a_y \Delta y_{\text{max}} = 0$$

$$\Delta y_{\text{max}} = -\frac{v_{yi}^2}{2a_y} = -\frac{v_i^2 (\sin \theta)^2}{2a_y} = -\frac{(23.0 \text{ m/s})^2 (\sin 25^\circ)^2}{(2)(-9.81 \text{ m/s}^2)} = \boxed{4.8 \text{ m}}$$

Givens

4. $\Delta x = 2.00 \text{ m}$
 $\Delta y = 0.55 \text{ m}$
 $\theta = 32.0^\circ$
 $a_y = -g = -9.81 \text{ m/s}^2$

Solutions

$$\Delta x = v_i(\cos \theta)\Delta t$$

$$\Delta t = \frac{\Delta x}{v_i(\cos \theta)}$$

$$\Delta y = v_i(\sin \theta)\Delta t + \frac{1}{2}a_y\Delta t^2 = v_i(\sin \theta)\left[\frac{\Delta x}{v_i(\cos \theta)}\right] + \frac{1}{2}a_y\left[\frac{\Delta x}{v_i(\cos \theta)}\right]^2$$

$$\Delta y = \Delta x(\tan \theta) + \frac{a_y\Delta x^2}{2v_i^2(\cos \theta)^2}$$

$$\Delta x(\tan \theta) - \Delta y = \frac{-a_y\Delta x^2}{2v_i^2(\cos \theta)^2}$$

$$2v_i^2(\cos \theta)^2 = \frac{-a_y\Delta x^2}{\Delta x(\tan \theta) - \Delta y}$$

$$v_i = \sqrt{\frac{-a_y\Delta x^2}{2(\cos \theta)^2[\Delta x(\tan \theta) - \Delta y]}}$$

$$v_i = \sqrt{\frac{(9.81 \text{ m/s}^2)(2.00 \text{ m})^2}{(2)(\cos 32.0^\circ)^2[(2.00 \text{ m})(\tan 32.0^\circ) - 0.55 \text{ m}]}}$$

$$v_i = \sqrt{\frac{(9.81 \text{ m/s}^2)(2.00 \text{ m})^2}{(2)(\cos 32.0^\circ)^2(1.25 - 0.55 \text{ m})}} = \sqrt{\frac{(9.81 \text{ m/s}^2)(2.00 \text{ m})^2}{(2)(\cos 32.0^\circ)^2(0.70 \text{ m})}} = \boxed{6.2 \text{ m/s}}$$

Two-Dimensional Motion and Vectors, Section 3 Review

2. $\Delta y = -125 \text{ m}$
 $v_x = 90.0 \text{ m/s}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta y = \frac{1}{2}a_y\Delta t^2$$

$$\Delta t = \frac{2\Delta y}{a_y} = \sqrt{\frac{(2)(-125 \text{ m})}{-9.81 \text{ m/s}^2}} = \boxed{5.05 \text{ s}}$$

$$\Delta x = v_x\Delta t = (90.0 \text{ m/s})(5.05 \text{ s}) = \boxed{454 \text{ m}}$$

3. $v_x = 30.0 \text{ m/s}$
 $\Delta y = -200.0 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$v_x = 30.0 \text{ m/s}$$

$$\Delta y = -200.0 \text{ m}$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

$$\Delta x = 192 \text{ m}$$

a. $\Delta y = \frac{1}{2}a_y\Delta t^2$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}}$$

$$\Delta x = v_x\Delta t = v_x\sqrt{\frac{2\Delta y}{a_y}} = (30.0 \text{ m/s})\sqrt{\frac{(2)(-200.0 \text{ m})}{-9.81 \text{ m/s}^2}} = \boxed{192 \text{ m}}$$

b. $v_y = \sqrt{v_{y,i}^2 + 2a_y\Delta y} = \sqrt{(0 \text{ m/s})^2 + (2)(-9.81 \text{ m/s}^2)(-200.0 \text{ m})} = \pm 62.6 \text{ m/s} = -62.6 \text{ m/s}$

$$v_{tot} = \sqrt{v_x^2 + v_y^2} = \sqrt{(30.0 \text{ m/s})^2 + (-62.6 \text{ m/s})^2} = \sqrt{9.00 \times 10^2 \text{ m}^2/\text{s}^2 + 3.92 \times 10^3 \text{ m}^2/\text{s}^2}$$

$$v_{tot} = \sqrt{4820 \text{ m}^2/\text{s}^2} = \boxed{69.4 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-62.6 \text{ m/s}}{30.0 \text{ m/s}}\right) = -64.4^\circ$$

$$\theta = \boxed{64.4^\circ \text{ below the horizontal}}$$

Two-Dimensional Motion and Vectors, Practice F

Givens

1. $\mathbf{v}_{te} = +15 \text{ m/s}$
 $\mathbf{v}_{bt} = -15 \text{ m/s}$

Solutions

$$\mathbf{v}_{be} = \mathbf{v}_{bt} + \mathbf{v}_{te} = -15 \text{ m/s} + 15 \text{ m/s} = \boxed{0 \text{ m/s}}$$

2. $\mathbf{v}_{aw} = +18.0 \text{ m/s}$
 $\mathbf{v}_{sa} = -3.5 \text{ m/s}$

$$\mathbf{v}_{sw} = \mathbf{v}_{sa} + \mathbf{v}_{aw} = -3.5 \text{ m/s} + 18.0 \text{ m/s}$$

$$\mathbf{v}_{sw} = \boxed{14.5 \text{ m/s in the direction that the aircraft carrier is moving}}$$

3. $\mathbf{v}_{fw} = 2.5 \text{ m/s north}$
 $\mathbf{v}_{we} = 3.0 \text{ m/s east}$

$$\mathbf{v}_{fe} = \mathbf{v}_{fw} + \mathbf{v}_{we}$$

$$v_{tot} = \sqrt{v_{fw}^2 + v_{we}^2} = \sqrt{(2.5 \text{ m/s})^2 + (3.0 \text{ m/s})^2}$$

$$v_{tot} = \sqrt{6.2 \text{ m}^2/\text{s}^2 + 9.0 \text{ m}^2/\text{s}^2} = \sqrt{15.2 \text{ m}^2/\text{s}^2} = \boxed{3.90 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{v_{fw}}{v_{we}}\right) = \tan^{-1}\left(\frac{2.5 \text{ m/s}}{3.0 \text{ m/s}}\right) = \boxed{(4.0 \times 10^1)^\circ \text{ north of east}}$$

4. $\mathbf{v}_{tr} = 25.0 \text{ m/s north}$
 $\mathbf{v}_{dt} = 1.75 \text{ m/s at } 35.0^\circ \text{ east of north}$

$$\mathbf{v}_{dr} = \mathbf{v}_{dt} + \mathbf{v}_{tr}$$

$$v_{x,tot} = v_{x,dt} = (1.75 \text{ m/s})(\sin 35.0^\circ) = 1.00 \text{ m/s}$$

$$v_{y,dt} = (1.75 \text{ m/s})(\cos 35.0^\circ) = 1.43 \text{ m/s}$$

$$v_{y,tot} = v_{tr} + v_{y,dt} = 25.0 \text{ m/s} + 1.43 \text{ m/s} = 26.4 \text{ m/s}$$

$$v_{tot} = \sqrt{(v_{x,tot})^2 + (v_{y,tot})^2} = \sqrt{(1.00 \text{ m/s})^2 + (26.4 \text{ m/s})^2}$$

$$v_{tot} = \sqrt{1.00 \text{ m}^2/\text{s}^2 + 697 \text{ m}^2/\text{s}^2} = \sqrt{698 \text{ m}^2/\text{s}^2} = \boxed{26.4 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{v_{x,tot}}{v_{y,tot}}\right) = \tan^{-1}\left(\frac{1.00 \text{ m/s}}{26.4 \text{ m/s}}\right) = \boxed{2.17^\circ \text{ east of north}}$$

Two-Dimensional Motion and Vectors, Section 4 Review

1. $\mathbf{v}_{wg} = -9 \text{ m/s}$
 $\mathbf{v}_{bg} = 1 \text{ m/s}$

$$\mathbf{v}_{bw} = \mathbf{v}_{bg} + \mathbf{v}_{gw} = \mathbf{v}_{bg} - \mathbf{v}_{wg} = (1 \text{ m/s}) - (-9 \text{ m/s}) = 1 \text{ m/s} + 9 \text{ m/s}$$

$$\mathbf{v}_{bw} = \boxed{10 \text{ m/s in the opposite direction}}$$

2. $\mathbf{v}_{bw} = 0.15 \text{ m/s north}$
 $\mathbf{v}_{we} = 1.50 \text{ m/s east}$

$$\mathbf{v}_{be} = \mathbf{v}_{bw} + \mathbf{v}_{we}$$

$$v_{tot} = \sqrt{v_{bw}^2 + v_{we}^2} = \sqrt{(0.15 \text{ m/s})^2 + (1.50 \text{ m/s})^2}$$

$$v_{tot} = \sqrt{0.022 \text{ m}^2/\text{s}^2 + 2.25 \text{ m}^2/\text{s}^2} = \sqrt{2.27 \text{ m}^2/\text{s}^2} = \boxed{1.51 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{v_{bw}}{v_{we}}\right) = \tan^{-1}\left(\frac{0.15 \text{ m/s}}{1.50 \text{ m/s}}\right) = \boxed{5.7^\circ \text{ north of east}}$$

Two-Dimensional Motion and Vectors, Chapter Review

Givens

6. $A = 3.00$ units (u)
 $B = -4.00$ units (u)

Solutions

Students should use graphical techniques.

a. $A + B = \sqrt{A^2 + B^2} = \sqrt{(3.00 \text{ u})^2 + (-4.00 \text{ u})^2}$

$$A + B = \sqrt{9.00 \text{ u}^2 + 16.0 \text{ u}^2} = \sqrt{25.0 \text{ u}^2} = \boxed{5.00 \text{ units}}$$

$$\theta = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{-4.00 \text{ u}}{3.00 \text{ u}}\right) = \boxed{53.1^\circ \text{ below the positive } x\text{-axis}}$$

b. $A - B = \sqrt{A^2 + (-B)^2} = \sqrt{(3.00 \text{ u})^2 + (4.00 \text{ u})^2}$

$$A - B = \sqrt{9.00 \text{ u}^2 + 16.0 \text{ u}^2} = \sqrt{25.0 \text{ u}^2} = \boxed{5.00 \text{ units}}$$

$$\theta = \tan^{-1}\left(\frac{-B}{A}\right) = \tan^{-1}\left(\frac{4.00 \text{ u}}{3.00 \text{ u}}\right) = \boxed{53.1^\circ \text{ above the positive } x\text{-axis}}$$

c. $A + 2B = \sqrt{A^2 + (2B)^2} = \sqrt{(3.00 \text{ u})^2 + (-8.00 \text{ u})^2}$

$$A + 2B = \sqrt{9.00 \text{ u}^2 + 64.0 \text{ u}^2} = \sqrt{73.0 \text{ u}^2} = \boxed{8.54 \text{ units}}$$

$$\theta = \tan^{-1}\left(\frac{2B}{A}\right) = \tan^{-1}\left(\frac{-8.00 \text{ u}}{3.00 \text{ u}}\right) = \boxed{69.4^\circ \text{ below the positive } x\text{-axis}}$$

d. $B - A = \sqrt{B^2 + (-A)^2} = \sqrt{(-4.00 \text{ u})^2 + (-3.00 \text{ u})^2} = \boxed{5.00 \text{ units}}$

$$\theta = \tan^{-1}\left(\frac{B}{-A}\right) = \tan^{-1}\left(\frac{-4.00 \text{ u}}{-3.00 \text{ u}}\right) = 53.1^\circ \text{ below the negative } x\text{-axis}$$

or $\boxed{127^\circ \text{ clockwise from the positive } x\text{-axis}}$

7. $A = 3.00$ m
 $B = 3.00$ m
 $\theta = 30.0^\circ$

Students should use graphical techniques.

$$A_x = A(\cos \theta) = (3.00 \text{ m})(\cos 30.0^\circ) = 2.60 \text{ m}$$

$$A_y = A(\sin \theta) = (3.00 \text{ m})(\sin 30.0^\circ) = 1.50 \text{ m}$$

a. $A + B = \sqrt{A_x^2 + (A_y + B)^2} = \sqrt{(2.60 \text{ m})^2 + (4.50 \text{ m})^2}$

$$A + B = \sqrt{6.76 \text{ m}^2 + 20.2 \text{ m}^2} = \sqrt{27.0 \text{ m}^2} = \boxed{5.20 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{A_y + B}{A_x}\right) = \tan^{-1}\left(\frac{4.50 \text{ m}}{2.60 \text{ m}}\right) = \boxed{60.0^\circ \text{ above the positive } x\text{-axis}}$$

b. $A - B = \sqrt{A_x^2 + (A_y - B)^2} = \sqrt{(2.60 \text{ m})^2 + (-1.50 \text{ m})^2}$

$$A - B = \sqrt{6.76 \text{ m}^2 + 2.25 \text{ m}^2} = \sqrt{9.01 \text{ m}^2} = \boxed{3.00 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{A_y - B}{A_x}\right) = \tan^{-1}\left(\frac{-1.50 \text{ m}}{2.60 \text{ m}}\right) = \boxed{30.0^\circ \text{ below the positive } x\text{-axis}}$$

$$\text{c. } B - A = \sqrt{(B - A_y)^2 + (-A_x)^2} = \sqrt{(1.50 \text{ m})^2 + (-2.60 \text{ m})^2}$$

$$B - A = \boxed{3.00 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{B - A_y}{-A_x}\right) = \tan^{-1}\left(\frac{1.50 \text{ m}}{-2.60 \text{ m}}\right) = 30.0^\circ \text{ above the negative } x\text{-axis}$$

or $\boxed{150^\circ}$ counterclockwise from the positive x -axis

$$\text{d. } A - 2B = \sqrt{A_x^2 + (A_y - 2B)^2} = \sqrt{(2.60 \text{ m})^2 + (-4.50 \text{ m})^2} = \boxed{5.20 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{A_y - 2B}{A_x}\right) = \tan^{-1}\left(\frac{-4.50 \text{ m}}{2.60 \text{ m}}\right) = \boxed{60.0^\circ \text{ below the positive } x\text{-axis}}$$

$$\text{8. } \Delta y_1 = -3.50 \text{ m}$$

$$d_2 = 8.20 \text{ m}$$

$$\theta_2 = 30.0^\circ$$

$$\Delta x_3 = -15.0 \text{ m}$$

Students should use graphical techniques.

$$\Delta x_2 = d_2(\cos \theta_2) = (8.20 \text{ m})(\cos 30.0^\circ) = 7.10 \text{ m}$$

$$\Delta y_2 = d_2(\sin \theta_2) = (8.20 \text{ m})(\sin 30.0^\circ) = 4.10 \text{ m}$$

$$\Delta x_{tot} = \Delta x_2 + \Delta x_3 = 7.10 \text{ m} - 15.0 \text{ m} = -7.9 \text{ m}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = -3.50 \text{ m} + 4.10 \text{ m} = 0.60 \text{ m}$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(-7.9 \text{ m})^2 + (0.60 \text{ m})^2}$$

$$d = \sqrt{62 \text{ m}^2 + 0.36 \text{ m}^2} = \sqrt{62 \text{ m}^2} = \boxed{7.9 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{0.60 \text{ m}}{-7.9 \text{ m}}\right) = \boxed{4.3^\circ \text{ north of west}}$$

$$\text{9. } \Delta x = -8.00 \text{ m}$$

$$\Delta y = 13.0 \text{ m}$$

Students should use graphical techniques.

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(-8.00 \text{ m})^2 + (13.0 \text{ m})^2}$$

$$d = \sqrt{64.0 \text{ m}^2 + 169 \text{ m}^2} = \sqrt{233 \text{ m}^2} = \boxed{15.3 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{13.0 \text{ m}}{-8.00 \text{ m}}\right) = \boxed{58.4^\circ \text{ south of east}}$$

$$\text{21. } \Delta x_1 = 3 \text{ blocks west}$$

$$= -3 \text{ blocks east}$$

$$\Delta y = 4 \text{ blocks north}$$

$$\Delta x_2 = 6 \text{ blocks east}$$

$$\text{a. } \Delta x_{tot} = \Delta x_1 + \Delta x_2 = -3 \text{ blocks} + 6 \text{ blocks} = 3 \text{ blocks}$$

$$\Delta y_{tot} = \Delta y = 4 \text{ blocks}$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(3 \text{ blocks})^2 + (4 \text{ blocks})^2}$$

$$d = \sqrt{9 \text{ blocks}^2 + 16 \text{ blocks}^2} = \sqrt{25 \text{ blocks}^2} = \boxed{5 \text{ blocks}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{4 \text{ blocks}}{3 \text{ blocks}}\right) = \boxed{53^\circ \text{ north of east}}$$

$$\text{b. distance traveled} = 3 \text{ blocks} + 4 \text{ blocks} + 6 \text{ blocks} = \boxed{13 \text{ blocks}}$$

Givens

22. $\Delta y_1 = -10.0$ yards

$\Delta x = 15.0$ yards

$\Delta y_2 = 50.0$ yards

Solutions

$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = -10.0$ yards + 50.0 yards = 40.0 yards

$\Delta x_{tot} = \Delta x = 15.0$ yards

$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(15.0 \text{ yards})^2 + (40.0 \text{ yards})^2}$

$d = \sqrt{225 \text{ yards}^2 + 1.60 \times 10^3 \text{ yards}^2} = \sqrt{1820 \text{ yards}^2} = \boxed{42.7 \text{ yards}}$

23. $\Delta y_1 = -40.0$ m

$\Delta x = \pm 15.0$ m

$\Delta y_2 = \pm 20.0$ m

Case 1: $\Delta y_{tot} = \Delta y_1 + \Delta y_2 = -40.0$ m - 20.0 m = -60.0 m

$\Delta x_{tot} = \Delta x = +15.0$ m

$d = \sqrt{(\Delta y_{tot})^2 + (\Delta x_{tot})^2} = \sqrt{(-60.0 \text{ m})^2 + (15.0 \text{ m})^2}$

$d = \sqrt{3.60 \times 10^3 \text{ m}^2 + 225 \text{ m}^2} = \sqrt{3820 \text{ m}^2} = \boxed{61.8 \text{ m}}$

$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{-60.0 \text{ m}}{15.0 \text{ m}}\right) = \boxed{76.0^\circ \text{ south of east}}$

Case 2: $\Delta y_{tot} = \Delta y_1 + \Delta y_2 = -40.0$ m + 20.0 m = -20.0 m

$\Delta x_{tot} = \Delta x = +15.0$ m

$d = \sqrt{(\Delta y_{tot})^2 + (\Delta x_{tot})^2} = \sqrt{(-20.0 \text{ m})^2 + (15.0 \text{ m})^2}$

$d = \sqrt{4.00 \times 10^2 \text{ m}^2 + 225 \text{ m}^2} = \sqrt{625 \text{ m}^2} = \boxed{25.0 \text{ m}}$

$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{-20.0 \text{ m}}{15.0 \text{ m}}\right) = \boxed{53.1^\circ \text{ south of east}}$

Case 3: $\Delta y_{tot} = \Delta y_1 + \Delta y_2 = -40.0$ m - 20.0 m = -60.0 m

$\Delta x_{tot} = \Delta x = -15.0$ m

$d = \sqrt{(\Delta y_{tot})^2 + (\Delta x_{tot})^2} = \sqrt{(-60.0 \text{ m})^2 + (-15.0 \text{ m})^2}$

$d = \boxed{61.8 \text{ m}}$

$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{-60.0 \text{ m}}{-15.0 \text{ m}}\right) = \boxed{76.0^\circ \text{ south of west}}$

Case 4: $\Delta y_{tot} = \Delta y_1 + \Delta y_2 = -40.0$ m + 20.0 m = -20.0 m

$\Delta x_{tot} = \Delta x = -15.0$ m

$d = \sqrt{(\Delta y_{tot})^2 + (\Delta x_{tot})^2} = \sqrt{(-20.0 \text{ m})^2 + (-15.0 \text{ m})^2}$

$d = \boxed{25.0 \text{ m}}$

$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{-20.0 \text{ m}}{-15.0 \text{ m}}\right) = \boxed{53.1^\circ \text{ south of west}}$

24. $d = 110.0$ m

$\theta = -10.0^\circ$

$\Delta x = d(\cos \theta) = (110.0 \text{ m})[\cos(-10.0^\circ)] = \boxed{108 \text{ m}}$

$\Delta y = d(\sin \theta) = (110.0 \text{ m})[\sin(-10.0^\circ)] = \boxed{-19.1 \text{ m}}$

25. $\theta = 25.0^\circ$

$d = 3.10$ km

$\Delta x = d(\cos \theta) = (3.10 \text{ km})(\cos 25.0^\circ) = \boxed{2.81 \text{ km east}}$

$\Delta y = d(\sin \theta) = (3.10 \text{ km})(\sin 25.0^\circ) = \boxed{1.31 \text{ km north}}$

Givens

26. $d_1 = 100.0 \text{ m}$
 $d_2 = 300.0 \text{ m}$
 $d_3 = 150.0 \text{ m}$
 $d_4 = 200.0 \text{ m}$
 $\theta_1 = 30.0^\circ$
 $\theta_2 = 60.0^\circ$

Solutions

$$\Delta x_{tot} = d_1 - d_3 \cos \theta_1 - d_4 \cos \theta_2$$

$$\Delta x_{tot} = 100.0 \text{ m} - (150.0 \text{ m})(\cos 30.0^\circ) - (200.0 \text{ m})(\cos 60.0^\circ)$$

$$\Delta x_{tot} = 100.0 \text{ m} - 1.30 \times 10^2 \text{ m} - 1.00 \times 10^2 \text{ m} = -1.30 \times 10^2 \text{ m}$$

$$\Delta y_{tot} = -d_2 - d_3 \sin \theta_1 + d_4 \sin \theta_2$$

$$\Delta y_{tot} = -300.0 \text{ m} - (150.0 \text{ m})(\sin 30.0^\circ) + (200.0 \text{ m})(\sin 60.0^\circ)$$

$$\Delta y_{tot} = -300.0 \text{ m} - 75.0 \text{ m} + 173 \text{ m} = -202 \text{ m}$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(1.30 \times 10^2 \text{ m})^2 + (-202 \text{ m})^2}$$

$$d = \sqrt{16\,900 \text{ m}^2 + 40\,800 \text{ m}^2} = \sqrt{57\,700 \text{ m}^2} = \boxed{2.40 \times 10^2 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{-202 \text{ m}}{-1.30 \times 10^2 \text{ m}}\right) = \boxed{57.2^\circ \text{ south of west}}$$

31. $\Delta y = -0.809 \text{ m}$
 $\Delta x = 18.3 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta y = \frac{1}{2}a_y \Delta t^2 = \frac{1}{2}a_y \left(\frac{\Delta x}{v_x}\right)^2$$

$$v_x = \sqrt{\frac{a_y}{2\Delta y}} \Delta x = \sqrt{\frac{-9.81 \text{ m/s}^2}{(2)(-0.809 \text{ m})}} (18.3 \text{ m}) = \boxed{45.1 \text{ m/s}}$$

32. $v_x = 18 \text{ m/s}$
 $\Delta y = -52 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta y = \frac{1}{2}a_y \Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{(2)(-52 \text{ m})}{-9.81 \text{ m/s}^2}} = \boxed{3.3 \text{ s}}$$

When the stone hits the water,

$$v_y = a_y \Delta t = (-9.81 \text{ m/s})(3.3 \text{ s}) = -32 \text{ m/s}$$

$$v_{tot} = \sqrt{v_x^2 + v_y^2} = \sqrt{(18 \text{ m/s})^2 + (-32 \text{ m/s})^2}$$

$$v_{tot} = \sqrt{320 \text{ m}^2/\text{s}^2 + 1000 \text{ m}^2/\text{s}^2} = \sqrt{1300 \text{ m}^2/\text{s}^2} = \boxed{36 \text{ m/s}}$$

33. $v_{x,s} = 15 \text{ m/s}$
 $v_{x,o} = 26 \text{ m/s}$
 $\Delta y = -5.0 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta y = \frac{1}{2}a_y \Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-5.0 \text{ m})}{-9.81 \text{ m/s}^2}} = 1.0 \text{ s}$$

$$\Delta x_s = v_{x,s} \Delta t = (15 \text{ m/s})(1.0 \text{ s}) = 15 \text{ m}$$

$$\Delta x_o = v_{x,o} \Delta t = (26 \text{ m/s})(1.0 \text{ s}) = 26 \text{ m}$$

$$\Delta x_o - \Delta x_s = 26 \text{ m} - 15 \text{ m} = \boxed{11 \text{ m}}$$

34. $v_i = 1.70 \times 10^3 \text{ m/s}$
 $\theta = 55.0^\circ$
 $a_y = -g = -9.81 \text{ m/s}^2$

a. $\Delta y = v_i(\sin \theta)\Delta t + \frac{1}{2}a_y \Delta t^2 = v_i(\sin \theta) + \frac{1}{2}a_y \Delta t = 0$

$$\Delta t = -\frac{2v_i(\sin \theta)}{a_y} = -\frac{(2)(1.70 \times 10^3 \text{ m/s})(\sin 55.0^\circ)}{-9.81 \text{ m/s}^2} = 284 \text{ s}$$

$$\Delta x = v_i(\cos \theta)\Delta t = (1.70 \times 10^3 \text{ m/s})(\cos 55.0^\circ)(284 \text{ s}) = \boxed{2.77 \times 10^5 \text{ m}}$$

b. $\Delta t = \boxed{284 \text{ s}}$ (See a.)

Givens

35. $\Delta x = 36.0 \text{ m}$

$$v_i = 20.0 \text{ m/s}$$

$$\theta = 53^\circ$$

$$\Delta y_{\text{bar}} = 3.05 \text{ m}$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

Solutions

a. $\Delta x = v_i(\cos \theta)\Delta t$

$$\Delta t = \frac{\Delta x}{v_i(\cos \theta)} = \frac{36.0 \text{ m}}{(20.0 \text{ m/s})(\cos 53^\circ)} = 3.0 \text{ s}$$

$$\Delta y = v_i(\sin \theta)\Delta t + \frac{1}{2}a_y\Delta t^2 = (20.0 \text{ m/s})(\sin 53^\circ)(3.0 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(3.0 \text{ s})^2$$

$$\Delta y = 48 \text{ m} - 44 \text{ m} = 4 \text{ m}$$

$$\Delta y = \Delta y_{\text{bar}} = 4 \text{ m} - 3.05 \text{ m} = 1 \text{ m}$$

The ball clears the goal by 1 m.

b. $v_{y,f} = v_i(\sin \theta) + a_y\Delta t = (20.0 \text{ m/s})(\sin 53^\circ) + (-9.81 \text{ m/s}^2)(3.0 \text{ s})$

$$v_{x,f} = 16 \text{ m/s} - 29 \text{ m/s} = -13 \text{ m/s}$$

The velocity of the ball as it passes over the crossbar is negative; therefore, the ball is falling.

36. $\Delta y = -1.00 \text{ m}$

$$\Delta x = 5.00 \text{ m}$$

$$\theta = 45.0^\circ$$

$$v = 2.00 \text{ m/s}$$

$$\Delta t = 0.329 \text{ s}$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

Find the initial velocity of the water when shot at rest horizontally 1 m above the ground.

$$\Delta y = \frac{1}{2}a_y\Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}}$$

$$\Delta x = v_x\Delta t$$

$$v_x = \frac{\Delta x}{\Delta t} = \frac{\Delta x}{\sqrt{\frac{2\Delta y}{a_y}}} = \frac{5.00 \text{ m}}{\sqrt{\frac{(2)(-1.00 \text{ m})}{-9.81 \text{ m/s}^2}}} = 11.1 \text{ m/s}$$

Find how far the water will go if it is shot horizontally 1 m above the ground while the child is sliding down the slide.

$$v_{x, \text{tot}} = v_x + v(\cos \theta)$$

$$\Delta x = v_{x, \text{tot}}\Delta t = [v_x + v(\cos \theta)]\Delta t = [11.1 \text{ m/s} + (2.00 \text{ m/s})(\cos 45.0^\circ)](0.329 \text{ s})$$

$$\Delta x = [11.1 \text{ m/s} + 1.41 \text{ m/s}](0.329 \text{ s}) = (12.5 \text{ m/s})(0.329 \text{ s}) = \boxed{4.11 \text{ m}}$$

37. $\Delta x_1 = 2.50 \times 10^3 \text{ m}$

$$\Delta x_2 = 6.10 \times 10^2 \text{ m}$$

$$\Delta y_{\text{mountain}} = 1.80 \times 10^3 \text{ m}$$

$$v_i = 2.50 \times 10^2 \text{ m/s}$$

$$\theta = 75.0^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

For projectile's full flight,

$$\Delta t = \frac{\Delta x}{v_i(\cos \theta)}$$

$$\Delta y = v_i(\sin \theta)\Delta t + \frac{1}{2}a_y\Delta t^2 = v_i(\sin \theta) + \frac{1}{2}a_y\Delta t = 0$$

$$v_i(\sin \theta) + \frac{1}{2}a_y\left[\frac{\Delta x}{v_i(\cos \theta)}\right] = 0$$

$$\Delta x = -\frac{2v_i^2(\sin \theta)(\cos \theta)}{a_y} = -\frac{(2)(2.50 \times 10^2 \text{ m/s})^2(\sin 75.0^\circ)(\cos 75.0^\circ)}{-9.81 \text{ m/s}^2} = 3190 \text{ m}$$

Distance between projectile and ship = $\Delta x - \Delta x_1 - \Delta x_2$

$$= 3190 \text{ m} - 2.50 \times 10^3 \text{ m} - 6.10 \times 10^2 \text{ m} = \boxed{80 \text{ m}}$$

For projectile's flight to the mountain,

$$\Delta t' = \frac{\Delta x_1}{v_i(\cos \theta)}$$

$$\Delta y = v_i(\sin \theta)\Delta t' + \frac{1}{2}a_y\Delta t'^2 = v_i(\sin \theta)\left[\frac{\Delta x_1}{v_i(\cos \theta)}\right] + \frac{1}{2}a_y\left[\frac{\Delta x_1}{v_i(\cos \theta)}\right]^2$$

Givens

Solutions

$$\Delta y = \Delta x_I (\tan \theta) + \frac{a_y \Delta x_I^2}{2v_i^2 (\cos \theta)^2}$$

$$\Delta y = (2.50 \times 10^3 \text{ m})(\tan 75.0^\circ) + \frac{(-9.81 \text{ m/s}^2)(2.50 \times 10^3 \text{ m})^2}{(2)(2.50 \times 10^2 \text{ m/s})^2 (\cos 75.0^\circ)^2}$$

$$\Delta y = 9330 \text{ m} - 7320 = 2010 \text{ m}$$

$$\text{distance above peak} = \Delta y - \Delta y_{\text{mountain}} = 2010 \text{ m} - 1.80 \times 10^3 \text{ m} = \boxed{210 \text{ m}}$$

43. $v_{re} = 1.50 \text{ m/s}$ east

$v_{br} = 10.0 \text{ m/s}$ north

$\Delta x = 325 \text{ m}$

a. $v_{be} = v_{br} + v_{re}$

$$v_{be} = \sqrt{v_{br}^2 + v_{re}^2} = \sqrt{(10.0 \text{ m/s})^2 + (1.50 \text{ m/s})^2}$$

$$v_{be} = \sqrt{1.00 \times 10^2 \text{ m}^2/\text{s}^2 + 2.25 \text{ m}^2/\text{s}^2} = \sqrt{102 \text{ m}^2/\text{s}^2} = \boxed{10.1 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{v_{re}}{v_{br}}\right) = \tan^{-1}\left(\frac{1.50 \text{ m/s}}{10.0 \text{ m/s}}\right) = \boxed{8.53^\circ \text{ east of north}}$$

b. $\Delta t = \frac{\Delta x}{v_{br}} = \frac{325 \text{ m}}{10.0 \text{ m/s}} = 32.5 \text{ s}$

$$\Delta y = v_{re} \Delta t = (1.50 \text{ m/s})(32.5 \text{ s}) = \boxed{48.8 \text{ m}}$$

44. $v_{we} = 50.0 \text{ km/h}$ south

$v_{aw} = 205 \text{ km/h}$

v_{ae} is directed due west

a. $v_{aw} = v_{ae} + (-v_{we})$

$$\frac{v_{we}}{v_{aw}} = \sin \theta$$

$$\theta = \sin^{-1}\left(\frac{v_{we}}{v_{aw}}\right) = \sin^{-1}\left(\frac{50.0 \text{ km/h}}{205 \text{ km/h}}\right) = \boxed{14.1^\circ \text{ north of west}}$$

b. $v_{aw}^2 = v_{ae}^2 + v_{we}^2$

$$v_{ae} = \sqrt{v_{aw}^2 - v_{we}^2} = \sqrt{(205 \text{ km/h})^2 - (50.0 \text{ km/h})^2}$$

$$v_{ae} = \sqrt{4.20 \times 10^4 \text{ km}^2/\text{h}^2 - 2.50 \times 10^3 \text{ km}^2/\text{h}^2}$$

$$v_{ae} = \sqrt{3.95 \times 10^4 \text{ km}^2/\text{h}^2} = \boxed{199 \text{ km/h}}$$

45. $\Delta x = 1.5 \text{ km}$

$v_{re} = 5.0 \text{ km/h}$

$v_{br} = 12 \text{ km/h}$

The boat's velocity in the x direction is greatest when the boat moves directly across the river with respect to the river.

$$\Delta t_{\min} = \frac{\Delta x}{v_{br}} = \frac{1.5 \text{ km}}{(12 \text{ km/h})(1 \text{ h}/60 \text{ min})} = \boxed{7.5 \text{ min}}$$

46. $v_{re} = 3.75 \text{ m/s}$ downstream

$v_{sr} = 9.50 \text{ m/s}$

v_{se} is directed across the river

a. $v_{sr} = v_{se} + (-v_{re})$

$$\frac{v_{re}}{v_{sr}} = \sin \theta$$

$$\theta = \sin^{-1}\left(\frac{3.75 \text{ m/s}}{9.50 \text{ m/s}}\right) = \boxed{23.2^\circ \text{ upstream from straight across}}$$

b. $v_{sr}^2 = v_{se}^2 + v_{re}^2$

$$v_{se} = \sqrt{v_{sr}^2 - v_{re}^2} = \sqrt{(9.50 \text{ m/s})^2 - (3.75 \text{ m/s})^2}$$

$$v_{se} = \sqrt{90.2 \text{ m}^2/\text{s}^2 - 14.1 \text{ m}^2/\text{s}^2} = \sqrt{76.1 \text{ m}^2/\text{s}^2} = 8.72 \text{ m/s}$$

$$v_{se} = \boxed{8.72 \text{ m/s directly across the river}}$$

Givens

47. $\Delta y = 21.0 \text{ m} - 1.0 \text{ m} = 20.0 \text{ m}$

$$\Delta x = 130.0 \text{ m}$$

$$\theta = 35.0^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

Solutions

a. $\Delta x = v_i \cos \theta \Delta t$ $\Delta t = \frac{\Delta x}{v_i \cos \theta}$

$$\Delta y = v_i \sin \theta \Delta t + \frac{1}{2} a_y (\Delta t)^2$$

$$\Delta y = v_i \sin \theta \left(\frac{\Delta x}{v_i \cos \theta} \right) + \frac{1}{2} a_y \left(\frac{\Delta x}{v_i \cos \theta} \right)^2$$

$$\Delta y = \Delta x \tan \theta + \frac{a_y (\Delta x)^2}{2 v_i^2 \cos^2 \theta}$$

$$v_i^2 = \frac{a_y (\Delta x)^2}{2 \cos^2 \theta (\Delta y - \Delta x \tan \theta)}$$

$$v_i = \frac{\Delta x}{\cos \theta} \sqrt{\frac{a_y}{2(\Delta y - \Delta x \tan \theta)}}$$

$$v_i = \frac{130.0 \text{ m}}{\cos 35.0^\circ} \sqrt{\frac{(-9.81 \text{ m/s}^2)}{2[(20.0 \text{ m}) - (130.0 \text{ m})(\tan 35.0^\circ)]}}$$

$$v_i = \boxed{41.7 \text{ m/s}}$$

b. $\Delta t = \frac{\Delta x}{v_i \cos \theta} = \frac{130.0 \text{ m}}{(41.7 \text{ m/s})(\cos 35.0^\circ)}$

$$\Delta t = \boxed{3.81 \text{ s}}$$

c. $v_{y,f} = v_i \sin \theta + a_y \Delta t = (41.7 \text{ m/s})(\sin 35.0^\circ) + (-9.81 \text{ m/s}^2)(3.81 \text{ s})$

$$v_{y,f} = 23.9 \text{ m/s} - 37.4 \text{ m/s} = \boxed{-13.5 \text{ m/s}}$$

$$v_{x,f} = v_i \cos \theta = (41.7 \text{ m/s})(\cos 35.0^\circ) = \boxed{34.2 \text{ m/s}}$$

$$v_f = \sqrt{1170 \text{ m}^2/\text{s}^2 + 182 \text{ m}^2/\text{s}^2} = \sqrt{1350 \text{ m}^2/\text{s}^2} = \boxed{36.7 \text{ m/s}}$$

48. $\Delta x = 12 \text{ m}$

$$\theta = 15^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

a. $\Delta x = v_i (\cos \theta) \Delta t$

$$\Delta t = \frac{\Delta x}{v_i (\cos \theta)}$$

$$\Delta y = v_i (\sin \theta) \Delta t + \frac{1}{2} a_y \Delta t^2 = v_i (\sin \theta) \left(\frac{\Delta x}{v_i (\cos \theta)} \right) + \frac{1}{2} a_y \left(\frac{\Delta x}{v_i (\cos \theta)} \right)^2 = 0$$

$$v_i (\sin \theta) + \frac{1}{2} a_y \left[\frac{\Delta x}{v_i (\cos \theta)} \right] = 0$$

$$2v_i^2 (\sin \theta) (\cos \theta) = -a_y \Delta x$$

$$v_i = \sqrt{\frac{-a_y \Delta x}{2(\sin \theta)(\cos \theta)}} = \sqrt{\frac{(9.81 \text{ m/s}^2)(12 \text{ m})}{(2)(\sin 15^\circ)(\cos 15^\circ)}} = \boxed{15 \text{ m/s}}$$

b. $\Delta t = \frac{\Delta x}{v_i (\cos \theta)} = \frac{12 \text{ m}}{(15 \text{ m/s})(\cos 15^\circ)} = 0.83 \text{ s}$

$$v_{y,f} = v_i (\sin \theta) + a_y \Delta t = (15 \text{ m/s})(\sin 15^\circ) + (-9.81 \text{ m/s}^2)(0.83 \text{ s})$$

$$v_{y,f} = 3.9 \text{ m/s} - 8.1 \text{ m/s} = -4.2 \text{ m/s}$$

$$v_{x,f} = v_x = v_i (\cos \theta) = (15 \text{ m/s})(\cos 15^\circ) = 14 \text{ m/s}$$

$$v_f = \sqrt{(v_{x,f})^2 + (v_{y,f})^2} = \sqrt{(14 \text{ m/s})^2 + (-4.2 \text{ m/s})^2}$$

$$v_f = \sqrt{2.0 \times 10^2 \text{ m}^2/\text{s}^2 + 18 \text{ m}^2/\text{s}^2} = \sqrt{220 \text{ m}^2/\text{s}^2} = \boxed{15 \text{ m/s}}$$

Givens

49. $\Delta x = 10.00 \text{ m}$

$\theta = 45.0^\circ$

$\Delta y = 3.05 \text{ m} - 2.00 \text{ m}$
 $= 1.05 \text{ m}$

$a_y = -g = -9.81 \text{ m/s}^2$

50. $\Delta x = 20.0 \text{ m}$

$\Delta t = 50.0 \text{ s}$

$v_{pe} = \pm 0.500 \text{ m/s}$

51. $\Delta y = -1.00 \text{ m}$

$\Delta x = 1.20 \text{ m}$

$a_y = -g = -9.81 \text{ m/s}^2$

52. $v_1 = 40.0 \text{ km/h}$

$v_2 = 60.0 \text{ km/h}$

$\Delta x_i = 125 \text{ m}$

Solutions

See the solution to problem 47 for a derivation of the following equation.

$$v_i = \sqrt{\frac{a_y \Delta x^2}{2(\cos \theta)^2[(\Delta y - \Delta x \tan \theta)]}} = \sqrt{\frac{(-9.81 \text{ m/s}^2)(10.00 \text{ m})^2}{(2)(\cos 45.0^\circ)^2[1.05 \text{ m} - (10.00 \text{ m})(\tan 45.0^\circ)]}$$

$$v_i = \sqrt{\frac{(-9.81 \text{ m/s}^2)(10.00 \text{ m})^2}{(2)(\cos 45.0^\circ)^2(1.05 \text{ m} - 10.00 \text{ m})}} = \sqrt{\frac{(-9.81 \text{ m/s}^2)(10.00 \text{ m})^2}{(2)(\cos 45.0^\circ)^2(-8.95 \text{ m})}} = \boxed{10.5 \text{ m/s}}$$

$$v_{eg} = \frac{\Delta x}{\Delta t} = \frac{20.0 \text{ m}}{50.0 \text{ s}} = 0.400 \text{ m/s}$$

$$\mathbf{v}_{pg} = \mathbf{v}_{pe} + \mathbf{v}_{eg}$$

a. Going up:

$$v_{pg} = v_{pe} + v_{eg} = 0.500 \text{ m/s} + 0.400 \text{ m/s} = 0.900 \text{ m/s}$$

$$\Delta t_{up} = \frac{\Delta x}{v_{pg}} = \frac{20.0 \text{ m}}{0.900 \text{ m/s}} = \boxed{22.2 \text{ s}}$$

b. Going down:

$$v_{pg} = -v_{pe} + v_{eg} = -0.500 \text{ m/s} + 0.400 \text{ m/s} = -0.100 \text{ m/s}$$

$$\Delta t_{down} = \frac{-\Delta x}{v_{pg}} = \frac{-20.0 \text{ m}}{-0.100 \text{ m/s}} = \boxed{2.00 \times 10^2 \text{ s}}$$

a. $\Delta x = v_x \Delta t$

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta y = \frac{1}{2} a_y \Delta t^2 = \frac{1}{2} a_y \left(\frac{\Delta x}{v_x} \right)^2 = \frac{a_y \Delta x^2}{2v_x^2}$$

$$v_x = \sqrt{\frac{a_y \Delta x^2}{2\Delta y}} = \sqrt{\frac{(-9.81 \text{ m/s}^2)(1.20 \text{ m})^2}{(2)(-1.00 \text{ m})}} = \boxed{2.66 \text{ m/s}}$$

b. The ball's velocity vector makes a 45° angle with the horizontal when $v_x = v_y$.

$$v_x = v_{yf} = a_y \Delta t$$

$$\Delta t = \frac{v_x}{a_y}$$

$$\Delta y = \frac{1}{2} a_y \Delta t^2 = \frac{1}{2} a_y \left(\frac{v_x}{a_y} \right)^2 = \frac{v_x^2}{2a_y}$$

$$\Delta y = \frac{(2.66 \text{ m/s})^2}{(2)(-9.81 \text{ m/s}^2)} = -0.361 \text{ m}$$

$$h = 1.00 \text{ m} - 0.361 \text{ m} = \boxed{0.64 \text{ m}}$$

For lead car:

$$\Delta x_{tot} = v_1 \Delta t + \Delta x_i$$

For chasing car:

$$\Delta x_{tot} = v_2 \Delta t$$

$$v_2 \Delta t = v_1 \Delta t + \Delta x_i$$

$$\Delta t = \frac{\Delta x_i}{v_2 - v_1} = \frac{(125 \text{ m})(10^{-3} \text{ km/m})}{(60.0 \text{ km/h} - 40.0 \text{ km/h})(1 \text{ h}/3600 \text{ s})}$$

$$\Delta t = \frac{125 \times 10^{-3} \text{ km}}{(20.0 \text{ km/h})(1 \text{ h}/3600 \text{ s})} = \boxed{22.5 \text{ s}}$$

Givens

53. $\theta = 60.0^\circ$

$$v_1 = 41.0 \text{ km/h}$$

$$v_2 = 25.0 \text{ km/h}$$

$$\Delta t_1 = 3.00 \text{ h}$$

$$\Delta t = 4.50 \text{ h}$$

Solutions

$$d_1 = v_1 \Delta t = (41.0 \text{ km/h})(3.00 \text{ h}) = 123 \text{ km}$$

$$\Delta x_1 = d_1 (\cos \theta) = (123 \text{ km})(\cos 60.0^\circ) = 61.5 \text{ km}$$

$$\Delta y_1 = d_1 (\sin \theta) = (123 \text{ km})(\sin 60.0^\circ) = 107 \text{ km}$$

$$\Delta t_2 = \Delta t - \Delta t_1 = 4.50 \text{ h} - 3.00 \text{ h} = 1.50 \text{ h}$$

$$\Delta y_2 = v_2 \Delta t_2 = (25.0 \text{ km/h})(1.50 \text{ h}) = 37.5 \text{ km}$$

$$\Delta x_{tot} = \Delta x_1 = 61.5 \text{ km}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 107 \text{ km} + 37.5 \text{ km} = 144 \text{ km}$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(61.5 \text{ km})^2 + (144 \text{ km})^2}$$

$$d = \sqrt{3780 \text{ km}^2 + 20700 \text{ km}^2} = \sqrt{24500 \text{ km}^2} = \boxed{157 \text{ km}}$$

54. $v_{bw} = \pm 7.5 \text{ m/s}$

$$v_{we} = 1.5 \text{ m/s}$$

$$\Delta x_d = 250 \text{ m}$$

$$\Delta x_u = -250 \text{ m}$$

$$\mathbf{v_{be}} = \mathbf{v_{bw}} + \mathbf{v_{we}}$$

Going downstream:

$$v_{be,d} = 7.5 \text{ m/s} + 1.5 \text{ m/s} = 9.0 \text{ m/s}$$

Going upstream:

$$v_{be,u} = -7.5 \text{ m/s} + 1.5 \text{ m/s} = -6.0 \text{ m/s}$$

$$\Delta t = \frac{\Delta x_d}{v_{be,d}} + \frac{\Delta x_u}{v_{be,u}} = \frac{250 \text{ m}}{9.0 \text{ m/s}} + \frac{-250 \text{ m}}{-6.0 \text{ m/s}} = 28 \text{ s} + 42 \text{ s} = \boxed{7.0 \times 10^1 \text{ s}}$$

55. $\theta = -24.0^\circ$

$$a = 4.00 \text{ m/s}^2$$

$$d = 50.0 \text{ m}$$

$$\Delta y = -30.0 \text{ m}$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

a. $d = \frac{1}{2} a \Delta t^2$

$$\Delta t_1 = \sqrt{\frac{2d}{a}} = \sqrt{\frac{(2)(50.0 \text{ m})}{4.00 \text{ m/s}^2}} = 5.00 \text{ s}$$

$$v_i = a \Delta t_1 = (4.00 \text{ m/s}^2)(5.00 \text{ s}) = 20.0 \text{ m/s}$$

$$v_{y,f} = \sqrt{v_i^2 (\sin \theta)^2 + 2a_y \Delta y} = \sqrt{(20.0 \text{ m/s})^2 [\sin(-24.0^\circ)]^2 + (2)(-9.81 \text{ m/s}^2)(-30.0 \text{ m})}$$

$$v_{y,f} = \sqrt{66.2 \text{ m}^2/\text{s}^2 + 589 \text{ m}^2/\text{s}^2} = \sqrt{655 \text{ m}^2/\text{s}^2} = \pm 25.6 \text{ m/s} = -25.6 \text{ m/s}$$

$$v_{y,f} = v_i (\sin \theta) + a_y \Delta t_2$$

$$\Delta t_2 = \frac{v_{y,f} - v_i (\sin \theta)}{a_y} = \frac{-25.6 \text{ m/s} - (20.0 \text{ m/s})(\sin -24.0^\circ)}{-9.81 \text{ m/s}^2}$$

$$\Delta t_2 = \frac{-25.6 \text{ m/s} + 8.13 \text{ m/s}}{-9.81 \text{ m/s}^2} = \frac{-17.5 \text{ m/s}}{-9.81 \text{ m/s}^2} = 1.78 \text{ s}$$

$$\Delta x = v_i (\cos \theta) \Delta t_2 = (20.0 \text{ m/s}) [\cos(-24.0^\circ)] (1.78 \text{ s}) = \boxed{32.5 \text{ m}}$$

b. $\Delta t_2 = \boxed{1.78 \text{ s}}$ (See **a.**)

Givens

56. $\theta = 34^\circ$

$$\Delta x = 240 \text{ m}$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

Solutions

a. $\Delta x = v_i(\cos \theta)\Delta t$

$$\Delta t = \frac{\Delta x}{v_i(\cos \theta)}$$

$$\Delta y = v_i(\sin \theta)\Delta t + \frac{1}{2}a_y\Delta t^2 = v_i(\sin \theta) + \frac{1}{2}a_y\Delta t = 0$$

$$v_i(\sin \theta) + \frac{1}{2}a_y \left[\frac{\Delta x}{v_i(\cos \theta)} \right] = v_i^2(\sin \theta) + \frac{a_y\Delta x}{2(\cos \theta)} = 0$$

$$v_i = \sqrt{\frac{-a_y\Delta x}{2(\cos \theta)(\sin \theta)}} = \sqrt{\frac{(9.81 \text{ m/s}^2)(240 \text{ m})}{(2)(\cos 34^\circ)(\sin 34^\circ)}} = \boxed{5.0 \times 10^1 \text{ m/s}}$$

b. $\Delta y_{\max} = \frac{v_{yf}^2 - v_{yi}^2}{2a_y}$

Because $v_{yf} = 0 \text{ m/s}$,

$$\Delta y_{\max} = \frac{-v_i^2(\sin \theta)^2}{2a_y} = \frac{-(5.0 \times 10^1 \text{ m/s})^2(\sin 34^\circ)^2}{(2)(-9.81 \text{ m/s}^2)}$$

$$\Delta y_{\max} = \boxed{4.0 \times 10^1 \text{ m}}$$

$$v_i(\sin \theta) \left[\frac{\Delta x}{2v_i(\cos \theta)} \right] + \frac{1}{2}a_y \left[\frac{\Delta x}{2v_i(\cos \theta)} \right]^2$$

$$\Delta y_{\max} = \frac{\Delta x}{2}(\tan \theta) + \frac{a_y\Delta x^2}{8v_i^2(\cos \theta)^2}$$

$$\Delta y_{\max} = \frac{(240 \text{ m})(\tan 34^\circ)}{2} + \frac{(-9.81 \text{ m/s}^2)(240 \text{ m})^2}{(8)(5.0 \times 10^1 \text{ m/s})^2(\cos 34^\circ)^2}$$

$$\Delta y_{\max} = 81 \text{ m} - 41 \text{ m} = \boxed{4.0 \times 10^1 \text{ m}}$$

57. $v_{ce} = 50.0 \text{ km/h}$ east

$$\theta = 60.0^\circ$$

a. $v_{ce} = v_{rc}(\sin \theta)$

$$v_{rc} = \frac{v_{ce}}{(\sin \theta)} = \frac{50.0 \text{ km/h}}{(\sin 60.0^\circ)} = 57.7 \text{ km/h}$$

$$\mathbf{v}_{rc} = \boxed{57.7 \text{ km/h at } 60.0^\circ \text{ west of the vertical}}$$

b. $v_{re} = v_{rc}(\cos \theta) = (57.7 \text{ km/h})(\cos 60.0^\circ) = 28.8 \text{ km/h}$

$$\mathbf{v}_{re} = \boxed{28.8 \text{ km/h straight down}}$$

58. $\Delta t_{\text{walk}} = 30.0 \text{ s}$

$$\Delta t_{\text{stand}} = 20.0 \text{ s}$$

$$v_{pe} = \frac{L}{\Delta t_{\text{walk}}} = \frac{L}{30.0 \text{ s}}$$

$$v_{eg} = \frac{L}{\Delta t_{\text{stand}}} = \frac{L}{20.0 \text{ s}}$$

$$\mathbf{v}_{pg} = \mathbf{v}_{pe} + \mathbf{v}_{eg}$$

$$v_{pg} = v_{pe} + v_{eg}$$

$$v_{pg} = \frac{L}{30.0 \text{ s}} + \frac{L}{20.0 \text{ s}} = \frac{2L + 3L}{60.0 \text{ s}} = \frac{5L}{60.0 \text{ s}}$$

$$\frac{L}{\Delta t} = \frac{5L}{60.0 \text{ s}}$$

$$\Delta t = \frac{60.0 \text{ s}}{5} = \boxed{12.0 \text{ s}}$$

Givens

59. $\Delta x_{\text{Earth}} = 3.0 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

Solutions

$$\Delta y = v_i(\sin \theta)\Delta t + \frac{1}{2}a_y\Delta t^2 = v_i(\sin \theta) + \frac{1}{2}a_y\Delta t = 0$$

$$\Delta t = -\frac{2v_i(\sin \theta)}{a_y}$$

$$\Delta x_{\text{Earth}} = v_i(\cos \theta)\Delta t = v_i(\cos \theta) \left[-\frac{2v_i(\sin \theta)}{a_y} \right]$$

$$\Delta x_{\text{Earth}} = -\frac{2v_i^2(\cos \theta)(\sin \theta)}{a_y} = \frac{2v_i^2(\cos \theta)(\sin \theta)}{g}$$

Because v_i and θ are the same for all locations,

$$\Delta x_{\text{Earth}} = \frac{k}{g}, \text{ where } k = 2v_i^2(\cos \theta)(\sin \theta)$$

$$k = g\Delta x_{\text{Earth}} = \left(\frac{g}{6}\right)\Delta x_{\text{moon}} = (0.38g)\Delta x_{\text{Mars}}$$

$$\Delta x_{\text{moon}} = 6\Delta x_{\text{Earth}} = (6)(3.0 \text{ m}) = \boxed{18 \text{ m}}$$

$$\Delta x_{\text{Mars}} = \frac{\Delta x_{\text{Earth}}}{0.38} = \frac{3.0 \text{ m}}{0.38} = \boxed{7.9 \text{ m}}$$

60. $v_x = 10.0 \text{ m/s}$
 $\theta = 60.0^\circ$
 $a_y = -g = -9.81 \text{ m/s}^2$

The observer on the ground sees the ball rise vertically, which indicates that the x -component of the ball's velocity is equal and opposite the velocity of the train.

$$v_x = v_i(\cos \theta)$$

$$v_i = \frac{v_x}{(\cos \theta)} = \frac{10.0 \text{ m/s}}{(\cos 60.0^\circ)} = 20.0 \text{ m/s}$$

At maximum height, $v_y = 0$, so

$$\Delta y_{\text{max}} = \frac{v_{yf}^2 - v_{yi}^2}{2a_y} = \frac{-v_i^2(\sin \theta)^2}{2a_y}$$

$$\Delta y_{\text{max}} = \frac{-(20.0 \text{ m/s})^2(\sin 60.0^\circ)^2}{(2)(-9.81 \text{ m/s}^2)} = \boxed{15.3 \text{ m}}$$

61. $v_i = 18.0 \text{ m/s}$
 $\theta = 35.0^\circ$
 $\Delta x_i = 18.0 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta y = v_i(\sin \theta)\Delta t + \frac{1}{2}a_y\Delta t^2 = v_i(\sin \theta) + \frac{1}{2}a_y\Delta t = 0$$

$$\Delta t = \frac{-2v_i(\sin \theta)}{a_y} = \frac{-2(18.0 \text{ m/s})(\sin 35.0^\circ)}{-9.81 \text{ m/s}^2} = 2.10 \text{ s}$$

$$\Delta x = v_i(\cos \theta)\Delta t = (18.0 \text{ m/s})(\cos 35.0^\circ)(2.10 \text{ s}) = 31.0 \text{ m}$$

$$\Delta x_{\text{run}} = \Delta x - \Delta x_i = 31.0 \text{ m} - 18.0 \text{ m} = 13.0 \text{ m}$$

$$v_{\text{run}} = \frac{\Delta x_{\text{run}}}{\Delta t} = \frac{13.0 \text{ m}}{2.10 \text{ s}} = \boxed{6.19 \text{ m/s downfield}}$$

62. $\theta = 53^\circ$
 $v_i = 75 \text{ m/s}$
 $\Delta t = 25 \text{ s}$
 $a = 25 \text{ m/s}^2$

$$a_y = a(\sin \theta) = (25 \text{ m/s}^2)(\sin 53^\circ) = 2.0 \times 10^1 \text{ m/s}^2$$

$$\Delta y = v_i(\sin \theta)\Delta t + \frac{1}{2}a_y\Delta t^2 = (75 \text{ m/s})(\sin 53^\circ)(25 \text{ s}) + \frac{1}{2}(2.0 \times 10^1 \text{ m/s}^2)(25 \text{ s})^2$$

$$\Delta y = 1500 \text{ m} + 6200 \text{ m} = 7700 \text{ m}$$

$$v_f = v_i + a\Delta t = 75 \text{ m/s} + (25 \text{ m/s}^2)(25 \text{ s}) = 75 \text{ m/s} + 620 \text{ m/s} = 7.0 \times 10^2 \text{ m/s}$$

Givens

$$v_i = v_f = 7.0 \times 10^2 \text{ m/s}$$

$$\theta = 53^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

$$v_i = 7.0 \times 10^2 \text{ m/s}$$

$$\theta = 53^\circ$$

Solutions

For the motion of the rocket after the boosters quit:

$$v_{y,f} = v_i(\sin \theta) + a_y \Delta t = 0$$

$$\Delta t = \frac{-v_i(\sin \theta)}{a_y} = \frac{-(7.0 \times 10^2 \text{ m/s})(\sin 53^\circ)}{-9.81 \text{ m/s}^2} = 57 \text{ s}$$

$$\Delta y = v_i(\sin \theta)\Delta t + \frac{1}{2}a_y\Delta t^2 = (7.0 \times 10^2 \text{ m/s})(\sin 53^\circ)(57 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(57 \text{ s})^2$$

$$\Delta y = 32\,000 \text{ m} - 16\,000 \text{ m} = 16\,000 \text{ m}$$

$$\text{a. } \Delta y_{\text{total}} = 7700 \text{ m} + 16\,000 \text{ m} = \boxed{2.4 \times 10^4 \text{ m}}$$

$$\text{b. } \Delta y = +\frac{1}{2}a_y\Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{(2)(-24\,000 \text{ m})}{-9.81 \text{ m/s}^2}} = 7.0 \times 10^1 \text{ s}$$

$$\Delta t_{\text{total}} = 25 \text{ s} + 57 \text{ s} + 7.0 \times 10^1 \text{ s} = \boxed{152 \text{ s}}$$

$$\text{c. } a_x = a(\cos \theta)$$

$$\Delta x = v_i(\cos \theta)\Delta t + \frac{1}{2}a_x\Delta t^2 = v_i(\cos \theta)\Delta t + \frac{1}{2}a(\cos \theta)\Delta t^2$$

$$\Delta x = (75 \text{ m/s})(\cos 53^\circ)(25 \text{ s}) + \frac{1}{2}(25 \text{ m/s}^2)(\cos 53^\circ)(25 \text{ s})^2$$

$$\Delta x = (1.1 \times 10^3 \text{ m}) + (4.7 \times 10^3 \text{ m}) = 5.8 \times 10^3 \text{ m}$$

After the rockets quit:

$$\Delta t = 57 \text{ s} + 7.0 \times 10^1 \text{ s} = 127 \text{ s}$$

$$\Delta x = v_i(\cos \theta)\Delta t = (7.0 \times 10^1 \text{ m/s})(\cos 53^\circ)(127 \text{ s}) = 5.4 \times 10^4 \text{ m}$$

$$\Delta x_{\text{tot}} = (5.8 \times 10^3 \text{ m}) + (5.4 \times 10^4 \text{ m}) = \boxed{6.0 \times 10^4 \text{ m}}$$

Two-Dimensional Motion and Vectors, Standardized Test Prep

$$\text{5. } \mathbf{v}_{\text{br}} = 5.0 \text{ m/s east}$$

$$\mathbf{v}_{\text{re}} = 5.0 \text{ m/s south}$$

$$\mathbf{v}_{\text{be}} = \mathbf{v}_{\text{br}} + \mathbf{v}_{\text{re}}$$

$$v_{\text{be}} = \sqrt{v_{\text{br}}^2 + v_{\text{re}}^2} = \sqrt{(5.0 \text{ m/s})^2 + (5.0 \text{ m/s})^2}$$

$$v_{\text{be}} = \sqrt{25 \text{ m}^2/\text{s}^2 + 25 \text{ m}^2/\text{s}^2} = \boxed{7.1 \text{ m/s}}$$

$$\text{6. } \Delta x = 125 \text{ m}$$

$$v_{\text{br}} = 5.0 \text{ m/s}$$

$$\Delta t = \frac{\Delta x}{v_{\text{br}}} = \frac{125 \text{ m}}{5.0 \text{ m/s}} = \boxed{25 \text{ s}}$$

$$\text{7. } \mathbf{v}_{\text{ap}} = 165 \text{ km/h south}$$

$$= -165 \text{ km/h north}$$

$$\mathbf{v}_{\text{pe}} = 145 \text{ km/h north}$$

$$\mathbf{v}_{\text{ae}} = \mathbf{v}_{\text{ap}} + \mathbf{v}_{\text{pe}}$$

$$\mathbf{v}_{\text{ae}} = -165 \text{ km/h north} + 145 \text{ km/h north} = -20 \text{ km/h north} = \boxed{20 \text{ km/h south}}$$

Givens

8. $\Delta x = 6.00 \text{ m}$
 $\Delta y = -5.40 \text{ m}$

Solutions

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(6.00 \text{ m})^2 + (-5.40 \text{ m})^2}$$

$$d = \sqrt{36.0 \text{ m}^2 + 29.2 \text{ m}^2} = \sqrt{65.2 \text{ m}^2} = \boxed{8.07 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{-5.40 \text{ m}}{6.00 \text{ m}}\right) = \boxed{42.0^\circ \text{ south of east}}$$

12. $v_{f,x} = v_x = 3.0 \text{ m/s}$
 $\Delta y = -1.5 \text{ m}$
 $g = -9.81 \text{ m/s}^2$
 $a_y = -g$

$$v_{f,y} = \sqrt{2a_y\Delta y} = \sqrt{-2g\Delta y} = \sqrt{(-2)(9.81 \text{ m/s}^2)(-1.5 \text{ m})} = 5.4 \text{ m/s}^2$$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(3.0 \text{ m/s})^2 + (5.4 \text{ m/s})^2} = \boxed{6.2 \text{ m/s}}$$

14. $d = 41.1 \text{ m}$
 $\theta = 40.0^\circ$

$$\Delta x = d(\cos \theta) = (41.1 \text{ m})(\cos 40.0^\circ) = \boxed{31.5 \text{ m}}$$

$$\Delta y = d(\sin \theta) = (41.1 \text{ m})(\sin 40.0^\circ) = \boxed{26.4 \text{ m}}$$

15. $\Delta t = 3.00 \text{ s}$
 $\theta = 30.0^\circ$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta y = v_i(\sin \theta)\Delta t + \frac{1}{2}a_y\Delta t^2 = v_i(\sin \theta) + \frac{1}{2}a_y\Delta t = 0$$

$$v_i = \frac{-a_y\Delta t}{2(\sin \theta)} = \frac{(9.81 \text{ m/s}^2)(3.00 \text{ s})}{(2)(\sin 30.0^\circ)} = \boxed{29.4 \text{ m/s}}$$

16. $v_i = 25.0 \text{ m/s}$
 $\theta = 45.0^\circ$
 $\Delta x = 50.0 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta x = v_i(\cos \theta)\Delta t$$

$$\Delta t = \frac{\Delta x}{v_i(\cos \theta)}$$

$$\Delta y = v_i(\sin \theta)\Delta t + \frac{1}{2}a_y\Delta t^2 = v_i(\sin \theta)\left[\frac{\Delta x}{v_i(\cos \theta)}\right] + \frac{1}{2}a_y\left[\frac{\Delta x}{v_i(\cos \theta)}\right]^2$$

$$\Delta y = \Delta x(\tan \theta) + \frac{a_y\Delta x^2}{2v_i^2(\cos \theta)^2} = (50.0 \text{ m})(\tan 45.0^\circ) + \frac{(-9.81 \text{ m/s}^2)(50.0 \text{ m})^2}{(2)(25.0 \text{ m/s})^2(\cos 45.0^\circ)^2}$$

$$\Delta y = 50.0 \text{ m} - 39.2 \text{ m} = \boxed{10.8 \text{ m}}$$

Two-Dimensional Motion and Vectors

Additional Practice A

Givens

1. $\Delta t_x = 7.95 \text{ s}$
 $\Delta y = 161 \text{ m}$
 $d = 226 \text{ m}$

Solutions

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\Delta x = \sqrt{d^2 - \Delta y^2} = \sqrt{(226 \text{ m})^2 - (161 \text{ m})^2} = \sqrt{5.11 \times 10^4 \text{ m}^2 - 2.59 \times 10^4 \text{ m}^2}$$

$$\Delta x = \sqrt{2.52 \times 10^4 \text{ m}^2} = 159 \text{ m}$$

$$\Delta x = \boxed{159 \text{ m}}$$

$$v = \frac{\Delta x}{\Delta t_x} = \frac{159 \text{ m}}{7.95 \text{ s}} = \boxed{20.0 \text{ m/s}}$$

2. $d_1 = 5.0 \text{ km}$
 $\theta_1 = 11.5^\circ$
 $d^2 = 1.0 \text{ km}$
 $\theta_2 = -90.0^\circ$

$$\Delta x_{tot} = d_1(\cos \theta_1) + d_2(\cos \theta_2) = (5.0 \text{ km})(\cos 11.5^\circ) + (1.0 \text{ km})[\cos(-90.0^\circ)]$$

$$\Delta x_{tot} = 4.9 \text{ km}$$

$$\Delta y_{tot} = d_1(\sin \theta_1) + d_2(\sin \theta_2) = (5.0 \text{ km})(\sin 11.5^\circ) + (1.0 \text{ km})[\sin(-90.0^\circ)]$$

$$= 1.0 \text{ km} - 1.0 \text{ km}$$

$$\Delta y_{tot} = 0.0 \text{ km}$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(4.9 \text{ km})^2 + (0.0 \text{ km})^2}$$

$$d = \boxed{4.9 \text{ km}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta y_{tot}}{\Delta x_{tot}} \right) = \tan^{-1} \left(\frac{0.0 \text{ km}}{4.9 \text{ km}} \right) = \boxed{0.0^\circ, \text{ or due east}}$$

3. $\Delta x = 5 \text{ jumps}$
 1 jump = 8.0 m
 $d = 68 \text{ m}$

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\Delta y = \sqrt{d^2 - \Delta x^2} = \sqrt{(68 \text{ m})^2 - [(5)(8.0 \text{ m})]^2} = \sqrt{4.6 \times 10^3 \text{ m}^2 - 1.6 \times 10^3 \text{ m}^2}$$

$$\Delta y = \sqrt{3.0 \times 10^3 \text{ m}^2} = 55 \text{ m}$$

$$\text{number of jumps northward} = \frac{55 \text{ m}}{8.0 \text{ m/jump}} = 6.9 \text{ jumps} = \boxed{7 \text{ jumps}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta x}{\Delta y} \right) = \tan^{-1} \left[\frac{(5)(8.0 \text{ m})}{55 \text{ m}} \right] = \boxed{36^\circ \text{ west of north}}$$

4. $\Delta x = 25.2 \text{ km}$
 $\Delta y = 21.3 \text{ km}$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(25.2 \text{ km})^2 + (21.3 \text{ km})^2}$$

$$d = \sqrt{635 \text{ km}^2 + 454 \text{ km}^2} = \sqrt{1089 \text{ km}^2}$$

$$d = \boxed{33.00 \text{ km}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta y}{\Delta x} \right) = \tan^{-1} \left(\frac{21.3 \text{ km}}{25.2 \text{ km}} \right)$$

$$\theta = \boxed{42.6^\circ \text{ south of east}}$$

Givens

5. $\Delta y = -483 \text{ m}$
 $\Delta x = 225 \text{ m}$

Solutions

$$\theta = \tan^{-1} \left(\frac{\Delta y}{\Delta x} \right) = \tan^{-1} \left(\frac{-483}{225} \right) = -65.0^\circ = \boxed{65.0^\circ \text{ below the waters surface}}$$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(225 \text{ m})^2 + (-483 \text{ m})^2}$$

$$d = \sqrt{5.06 \times 10^4 \text{ m}^2 + 2.33 \times 10^5 \text{ m}^2} = \sqrt{2.84 \times 10^5 \text{ m}^2}$$

$$d = \boxed{533 \text{ m}}$$

6. $v = 15.0 \text{ m/s}$
 $\Delta t_x = 8.0 \text{ s}$
 $d = 180.0 \text{ m}$

$$d^2 = \Delta x^2 + \Delta y^2 = (v\Delta t_x)^2 + (v\Delta t_y)^2$$

$$d^2 = v^2(\Delta t_x^2 + \Delta t_y^2)$$

$$\Delta t_y = \sqrt{\left(\frac{d}{v}\right)^2 - \Delta t_x^2} = \sqrt{\left(\frac{180.0 \text{ m}}{15.0 \text{ m/s}}\right)^2 - (8.0 \text{ s})^2} = \sqrt{144 \text{ s}^2 - 64 \text{ s}^2} = \sqrt{8.0 \times 10^1 \text{ s}^2}$$

$$\Delta t_y = \boxed{8.9 \text{ s}}$$

7. $v = 8.00 \text{ km/h}$
 $\Delta t_x = 15.0 \text{ min}$
 $\Delta t_y = 22.0 \text{ min}$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(v\Delta t_x)^2 + (v\Delta t_y)^2}$$

$$= v\sqrt{\Delta t_x^2 + \Delta t_y^2}$$

$$d = (8.00 \text{ km/h}) \left(\frac{1 \text{ h}}{60 \text{ min}} \right) \sqrt{(15.0 \text{ min})^2 + (22.0 \text{ min})^2}$$

$$d = (8.00 \text{ km/h}) \left(\frac{1 \text{ h}}{60 \text{ min}} \right) \sqrt{225 \text{ min}^2 + 484 \text{ min}^2}$$

$$d = \left(\frac{8.00 \text{ km}}{60 \text{ min}} \right) \sqrt{709 \text{ min}^2} = \boxed{3.55 \text{ km}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta y}{\Delta x} \right) = \tan^{-1} \left(\frac{v\Delta t_y}{v\Delta t_x} \right) = \tan^{-1} \left(\frac{\Delta t_y}{\Delta t_x} \right) = \tan^{-1} \left(\frac{22.0 \text{ min}}{15.0 \text{ min}} \right)$$

$$\theta = \boxed{55.7^\circ \text{ north of east}}$$

Additional Practice B

1. $d = (5)(33.0 \text{ cm})$
 $\Delta y = 88.0 \text{ cm}$

$$\theta = \sin^{-1} \left(\frac{\Delta y}{d} \right) = \sin^{-1} \left[\frac{88.0 \text{ cm}}{(5)(33.0 \text{ cm})} \right] = \boxed{32.2^\circ \text{ north of west}}$$

$$\Delta x = d(\cos \theta) = (5)(33.0 \text{ cm})(\cos 32.2^\circ) = \boxed{1.40 \times 10^2 \text{ cm to the west}}$$

2. $\theta = 60.0^\circ$
 $d = 10.0 \text{ m}$

$$\Delta x = d(\cos \theta) = (10.0 \text{ m})(\cos 60.0^\circ) = \boxed{5.00 \text{ m}}$$

$$\Delta y = d(\sin \theta) = (10.0 \text{ m})(\sin 60.0^\circ) = \boxed{8.66 \text{ m}}$$

3. $d = 10.3 \text{ m}$
 $\Delta y = -6.10 \text{ m}$

Finding the angle between d and the x -axis yields,

$$\theta_1 = \sin^{-1} \left(\frac{\Delta y}{d} \right) = \sin^{-1} \left(\frac{-6.10 \text{ m}}{10.3 \text{ m}} \right) = -36.3^\circ$$

The angle between d and the negative y -axis is therefore,

$$\theta = -90.0 - (-36.3^\circ) = -53.7^\circ$$

$$\theta = \boxed{53.7^\circ \text{ on either side of the negative } y\text{-axis}}$$

$$d^2 + \Delta x^2 + \Delta y^2$$

$$\Delta x = \sqrt{d^2 - \Delta y^2} = \sqrt{(10.3 \text{ m})^2 - (-6.10 \text{ m})^2} = \sqrt{106 \text{ m}^2 - 37.2 \text{ m}^2} = \sqrt{69 \text{ m}^2}$$

$$\Delta x = \boxed{\pm 8.3 \text{ m}}$$

Givens

4. $d = (8)(4.5 \text{ m})$
 $\theta = 35^\circ$

Solutions

$$\Delta x = d(\cos \theta) = (8)(4.5 \text{ m})(\cos 35^\circ) = \boxed{29 \text{ m}}$$

$$\Delta y = d(\sin \theta) = (8)(4.5 \text{ m})(\sin 35^\circ) = \boxed{21 \text{ m}}$$

5. $v = 347 \text{ km/h}$
 $\theta = 15.0^\circ$

$$v_x = v(\cos \theta) = (347 \text{ km/h})(\cos 15.0^\circ) = \boxed{335 \text{ km/h}}$$

$$v_y = v(\sin \theta) = (347 \text{ km/h})(\sin 15.0^\circ) = \boxed{89.8 \text{ km/h}}$$

6. $v = 372 \text{ km/h}$
 $\Delta t = 8.7 \text{ s}$
 $\theta = 60.0^\circ$

$$d = v\Delta t = (372 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(10^3 \text{ m/km})(8.7 \text{ s}) = 9.0 \times 10^2 \text{ m}$$

$$\Delta x = d(\cos \theta) = (9.0 \times 10^2 \text{ m})(\cos 60.0^\circ) = \boxed{450 \text{ m east}}$$

$$\Delta y = d(\sin \theta) = (9.0 \times 10^2 \text{ m})(\sin 60.0^\circ) = \boxed{780 \text{ m north}}$$

7. $d = 14\,890 \text{ km}$
 $\theta = 25.0^\circ$
 $\Delta t = 18.5 \text{ h}$

$$v_{avg} = \frac{d}{\Delta t} = \frac{1.489 \times 10^4 \text{ km}}{18.45 \text{ h}} = \boxed{805 \text{ km/h}}$$

$$v_x = v_{avg}(\cos \theta) = (805 \text{ km/h})(\cos 25.0^\circ) = \boxed{730 \text{ km/h east}}$$

$$v_y = v_{avg}(\sin \theta) = (805 \text{ km/h})(\sin 25.0^\circ) = \boxed{340 \text{ km/h south}}$$

8. $v_i = 6.0 \times 10^2 \text{ km/h}$
 $v_f = 2.3 \times 10^3 \text{ km/h}$
 $\Delta t = 120 \text{ s}$
 $\theta = 35^\circ$ with respect to horizontal

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

$$a = \frac{(2.3 \times 10^3 \text{ km/h} - 6.0 \times 10^2 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(10^3 \text{ m/km})}{1.2 \times 10^2 \text{ s}}$$

$$a = \frac{(1.7 \times 10^3 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(10^3 \text{ m/km})}{1.2 \times 10^2 \text{ s}}$$

$$a = 3.9 \text{ m/s}^2$$

$$a_x = a(\cos \theta) = (3.9 \text{ m/s}^2)(\cos 35^\circ) = \boxed{3.2 \text{ m/s}^2 \text{ horizontally}}$$

$$a_y = a(\sin \theta) = (3.9 \text{ m/s}^2)(\sin 35^\circ) = \boxed{2.2 \text{ m/s}^2 \text{ vertically}}$$

Additional Practice C

1. $\Delta x_1 = 250.0 \text{ m}$
 $d_2 = 125.0 \text{ m}$
 $\theta_2 = 120.0^\circ$

$$\Delta x_2 = d_2(\cos \theta_2) = (125.0 \text{ m})(\cos 120.0^\circ) = -62.50 \text{ m}$$

$$\Delta y_2 = d_2(\sin \theta_2) = (125.0 \text{ m})(\sin 120.0^\circ) = 108.3 \text{ m}$$

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 250.0 \text{ m} - 62.50 \text{ m} = 187.5 \text{ m}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 0 \text{ m} + 108.3 \text{ m} = 108.3 \text{ m}$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(187.5 \text{ m})^2 + (108.3 \text{ m})^2}$$

$$d = \sqrt{3.516 \times 10^4 \text{ m}^2 + 1.173 \times 10^4 \text{ m}^2} = \sqrt{4.689 \times 10^4 \text{ m}^2}$$

$$d = \boxed{216.5 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{108.3 \text{ m}}{187.5 \text{ m}}\right) = \boxed{30.01^\circ \text{ north of east}}$$

Givens

$$2. \nu = 3.53 \times 10^3 \text{ km/h}$$

$$\Delta t_1 = 20.0 \text{ s}$$

$$\Delta t_2 = 10.0 \text{ s}$$

$$\theta_1 = 15.0^\circ$$

$$\theta_2 = 35.0^\circ$$

Solutions

$$\Delta x_1 = \nu \Delta t_1 (\cos \theta_1)$$

$$\Delta x_1 = (3.53 \times 10^3 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) (10^3 \text{ m/km}) (20.0 \text{ s}) (\cos 15.0^\circ) = 1.89 \times 10^4 \text{ m}$$

$$\Delta y_1 = \nu \Delta t_1 (\sin \theta_1)$$

$$\Delta y_1 = (3.53 \times 10^3 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) (10^3 \text{ m/km}) (20.0 \text{ s}) (\sin 15.0^\circ) = 5.08 \times 10^3 \text{ m}$$

$$\Delta x_2 = \nu \Delta t_2 (\cos \theta_2)$$

$$\Delta x_2 = (3.53 \times 10^3 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) (10^3 \text{ m/km}) (10.0 \text{ s}) (\cos 35.0^\circ) = 8.03 \times 10^3 \text{ m}$$

$$\Delta y_2 = \nu \Delta t_2 (\sin \theta_2)$$

$$\Delta y_2 = (3.53 \times 10^3 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) (10^3 \text{ m/km}) (10.0 \text{ s}) (\sin 35.0^\circ) = 5.62 \times 10^3 \text{ m}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 5.08 \times 10^3 \text{ m} + 5.62 \times 10^3 \text{ m} = \boxed{1.07 \times 10^4 \text{ m}}$$

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 1.89 \times 10^4 \text{ m} + 8.03 \times 10^3 \text{ m} = \boxed{2.69 \times 10^4 \text{ m}}$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(2.69 \times 10^4 \text{ m})^2 + (1.07 \times 10^4 \text{ m})^2}$$

$$d = \sqrt{7.24 \times 10^8 \text{ m}^2 + 1.11 \times 10^8 \text{ m}^2} = \sqrt{8.35 \times 10^8 \text{ m}^2}$$

$$d = \boxed{2.89 \times 10^4 \text{ m}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta y_{tot}}{\Delta x_{tot}} \right) = \tan^{-1} \left(\frac{1.07 \times 10^4 \text{ m}}{2.69 \times 10^4 \text{ m}} \right)$$

$$\theta = \boxed{21.7^\circ \text{ above the horizontal}}$$

$$3. \Delta x_1 + \Delta x_2 = 2.00 \times 10^2 \text{ m}$$

$$\Delta y_1 + \Delta y_2 = 0$$

$$\theta_1 = 30.0^\circ$$

$$\theta_2 = -45.0^\circ$$

$$\nu = 11.6 \text{ km/h}$$

$$\Delta y_1 = d_1 (\sin \theta_1) = -\Delta y_2 = -d_2 (\sin \theta_2)$$

$$d_1 = -d_2 \left(\frac{\sin \theta_2}{\sin \theta_1} \right) = -d_2 \left[\frac{\sin(-45.0^\circ)}{\sin 30.0^\circ} \right] = 1.41 d_2$$

$$\Delta x_1 = d_1 (\cos \theta_1) = (1.41 d_2) (\cos 30.0^\circ) = 1.22 d_2$$

$$\Delta x_2 = d_2 (\cos \theta_2) = d_2 [\cos(-45.0^\circ)] = 0.707 d_2$$

$$\Delta x_1 + \Delta x_2 = d_2 (1.22 + 0.707) = 1.93 d_2 = 2.00 \times 10^2 \text{ m}$$

$$d_2 = \boxed{104 \text{ m}}$$

$$d_1 = (1.41) d_2 = (1.41)(104 \text{ m}) = \boxed{147 \text{ m}}$$

$$\nu = 11.6 \text{ km/h} = (11.6 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) (10^3 \text{ m/km}) = 3.22 \text{ m/s}$$

$$\Delta t_1 = \frac{d_1}{\nu} = \left(\frac{147 \text{ m}}{3.22 \text{ m/s}} \right) = 45.7 \text{ s}$$

$$\Delta t_2 = \frac{d_2}{\nu} = \left(\frac{104 \text{ m}}{3.22 \text{ m/s}} \right) = 32.3 \text{ s}$$

$$\Delta t_{tot} = \Delta t_1 + \Delta t_2 = 45.7 \text{ s} + 32.3 \text{ s} = \boxed{78.0 \text{ s}}$$

Givens

4. $v = 925 \text{ km/h}$
 $\Delta t_1 = 1.50 \text{ h}$
 $\Delta t_2 = 2.00 \text{ h}$
 $\theta_2 = 135^\circ$

Solutions

$$\begin{aligned}d_1 &= v\Delta t_1 = (925 \text{ km/h})(10^3 \text{ m/km})(1.50 \text{ h}) = 1.39 \times 10^6 \text{ m} \\d_2 &= v\Delta t_2 = (925 \text{ km/h})(10^3 \text{ m/km})(2.00 \text{ h}) = 1.85 \times 10^6 \text{ m} \\ \Delta x_1 &= d_1 = 1.39 \times 10^6 \text{ m} \\ \Delta y_1 &= 0 \text{ m} \\ \Delta x_2 &= d_2(\cos \theta_2) = (1.85 \times 10^6 \text{ m})(\cos 135^\circ) = -1.31 \times 10^6 \text{ m} \\ \Delta y_2 &= d_2(\sin \theta_2) = (1.85 \times 10^6 \text{ m})(\sin 135^\circ) = 1.31 \times 10^6 \text{ m} \\ \Delta x_{tot} &= \Delta x_1 + \Delta x_2 = 1.39 \times 10^6 \text{ m} + (-1.31 \times 10^6 \text{ m}) = 0.08 \times 10^6 \text{ m} \\ \Delta y_{tot} &= \Delta y_1 + \Delta y_2 = 0 \text{ m} + 1.31 \times 10^6 \text{ m} = 1.31 \times 10^6 \text{ m} \\ d &= \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(0.08 \times 10^6 \text{ m})^2 + (1.31 \times 10^6 \text{ m})^2} \\ d &= \sqrt{6 \times 10^9 \text{ m}^2 + 1.72 \times 10^{12} \text{ m}^2} = \sqrt{1.73 \times 10^{12} \text{ m}^2} \\ d &= \boxed{1.32 \times 10^6 \text{ m} = 1.32 \times 10^3 \text{ km}} \\ \theta &= \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{1.31 \times 10^6 \text{ m}}{0.08 \times 10^6 \text{ m}}\right) = 86.5^\circ = 90.0^\circ - 3.5^\circ \\ \theta &= \boxed{3.5^\circ \text{ east of north}}\end{aligned}$$

5. $v = 57.2 \text{ km/h}$
 $\Delta t_1 = 2.50 \text{ h}$
 $\Delta t_2 = 1.50 \text{ h}$
 $\theta_2 = 30.0^\circ$

$$\begin{aligned}d_1 &= v\Delta t_1 = (57.2 \text{ km/h})(2.50 \text{ h}) = 143 \text{ km} \\d_2 &= v\Delta t_2 = (57.2 \text{ km/h})(1.50 \text{ h}) = 85.8 \text{ km} \\ \Delta x_{tot} &= d_1 + d_2(\cos \theta_2) = 143 \text{ km} + (85.8 \text{ km})(\cos 30.0^\circ) = 143 \text{ km} + 74.3 \text{ km} = 217 \text{ km} \\ \Delta y_{tot} &= d_2(\sin \theta_2) = (85.8 \text{ km})(\sin 30.0^\circ) = 42.9 \text{ km} \\ d &= \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(217 \text{ km})^2 + (42.9 \text{ km})^2} \\ d &= \sqrt{4.71 \times 10^4 \text{ km}^2 + 1.84 \times 10^3 \text{ km}^2} = \sqrt{4.89 \times 10^4 \text{ km}^2} \\ d &= \boxed{221 \text{ km}} \\ \theta &= \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{42.9 \text{ km}}{217 \text{ km}}\right) = \boxed{11.2^\circ \text{ north of east}}\end{aligned}$$

Additional Practice D

1. $v_x = 9.37 \text{ m/s}$
 $\Delta y = -2.00 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\begin{aligned}\Delta t &= \sqrt{\frac{2\Delta y}{a_y}} = \frac{\Delta x}{v_x} \\ \Delta x &= v_x \sqrt{\frac{2\Delta y}{a_y}} = (9.37 \text{ m/s}) \sqrt{\frac{(2)(-2.00 \text{ m})}{(-9.81 \text{ m/s}^2)}} = 5.98 \text{ m}\end{aligned}$$

The river is 5.98 m wide.

2. $\Delta x = 7.32 \text{ km}$
 $\Delta y = -8848 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\begin{aligned}\Delta t &= \sqrt{\frac{2\Delta y}{a_y}} = \frac{\Delta x}{v_x} \\ v_x &= \sqrt{\frac{a_y}{2\Delta y}} \Delta x = \sqrt{\frac{(-9.81 \text{ m/s}^2)}{(2)(-8848 \text{ m})}} (7.32 \times 10^3 \text{ m}) = \boxed{172 \text{ m/s}}\end{aligned}$$

No. The arrow must have a horizontal speed of 172 m/s, which is much greater than 100 m/s.

Givens

3. $\Delta x = 471 \text{ m}$
 $v_i = 80.0 \text{ m/s}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta y = \frac{1}{2} a_y (\Delta t)^2 = \frac{a_y (\Delta x)^2}{2v_x^2} = \frac{(-9.81 \text{ m/s}^2)(471 \text{ m})^2}{(2)(80.0 \text{ m/s})^2} = -1.70 \times 10^2 \text{ m}$$

The cliff is $1.70 \times 10^2 \text{ m}$ high.

4. $v_x = 372 \text{ km/h}$
 $\Delta x = 40.0 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta y = \frac{1}{2} a_y (\Delta t)^2 = \frac{a_y (\Delta x)^2}{2v_x^2} = \frac{(-9.81 \text{ m/s}^2)(40.0 \text{ m})^2}{(2) \left[(372 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right) \right]^2}$$

$$\Delta y = -0.735 \text{ m}$$

The ramp is 0.735 m above the ground.

5. $\Delta x = 25 \text{ m}$
 $v_x = 15 \text{ m/s}$
 $a_y = -g = -9.81 \text{ m/s}^2$
 $h = 25 \text{ m}$

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta y = \frac{1}{2} a_y (\Delta t)^2 = \frac{a_y (\Delta x)^2}{2v_x^2} = \frac{(-9.81 \text{ m/s}^2)(25 \text{ m})^2}{(2)(15 \text{ m/s})^2}$$

$$\Delta y = h - h' = -14 \text{ m}$$

$$h' = h - \Delta y = 25 \text{ m} - (-14 \text{ m})$$

$$= \boxed{39 \text{ m}}$$

6. $\ell = 420 \text{ m}$
 $\Delta y = \frac{-\ell}{2}$
 $\Delta x = \ell$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \frac{\Delta x}{v_x}$$

$$v_x = \sqrt{\frac{a_y}{2\Delta y}} \Delta x = \sqrt{\frac{(-9.81 \text{ m/s}^2)}{(2)(-210 \text{ m})}} (420 \text{ m}) = \boxed{64 \text{ m/s}}$$

7. $\Delta y = -2.45 \text{ m}$
 $v = 12.0 \text{ m/s}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$v_y^2 = 2a_y \Delta y$$

$$v^2 = v_x^2 + v_y^2 = v_x^2 + 2a_y \Delta y$$

$$v_x = \sqrt{v^2 - 2a_y \Delta y} = \sqrt{(12.0 \text{ m/s})^2 - (2)(-9.81 \text{ m/s}^2)(-2.45 \text{ m})}$$

$$v_x = \sqrt{144 \text{ m}^2/\text{s}^2 - 48.1 \text{ m}^2/\text{s}^2}$$

$$= \sqrt{96 \text{ m}^2/\text{s}^2}$$

$$v_x = \boxed{9.8 \text{ m/s}}$$

Givens

8. $\Delta y = -1.95 \text{ m}$

$$v_x = 3.0 \text{ m/s}$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

Solutions

$$v_y^2 = 2a_y \Delta y$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_x^2 + 2a_y \Delta y}$$

$$v = \sqrt{(3.0 \text{ m/s})^2 + (2)(-9.81 \text{ m/s}^2)(-1.95 \text{ m})}$$

$$v = \sqrt{9.0 \text{ m}^2/\text{s}^2 + 38.3 \text{ m}^2/\text{s}^2} = \sqrt{47.3 \text{ m}^2/\text{s}^2} = \boxed{6.88 \text{ m/s}}$$

$$\theta = \tan^{-1} \left(\frac{v_y}{v_x} \right) = \tan^{-1} \left(\frac{\sqrt{2a_y \Delta y}}{v_x} \right) = \tan^{-1} \left(\frac{\sqrt{(2)(-9.81 \text{ m/s}^2)(-1.95 \text{ m})}}{3.0 \text{ m/s}} \right)$$

$$\theta = \boxed{64^\circ \text{ below the horizontal}}$$

Additional Practice E

1. $\Delta x = 201.24 \text{ m}$

$$\theta = 35.0^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

$$\Delta y = v_i (\sin \theta) \Delta t + \frac{1}{2} a_y (\Delta t)^2 = v_i (\sin \theta) \Delta t + \frac{1}{2} a_y \Delta t^2 = 0$$

$$\Delta x = v_i (\cos \theta) \Delta t$$

$$\Delta t = \frac{\Delta x}{v_i (\cos \theta)}$$

$$v_i (\sin \theta) = -\frac{1}{2} a_y \left[\frac{\Delta x}{v_i (\cos \theta)} \right]$$

$$v_i = \sqrt{\frac{-a_y \Delta x}{2 (\sin \theta) (\cos \theta)}} = \sqrt{\frac{-(-9.81 \text{ m/s}^2)(201.24 \text{ m})}{(2)(\sin 35.0^\circ)(\cos 35.0^\circ)}}$$

$$v_i = \boxed{45.8 \text{ m/s}}$$

2. $\Delta x = 9.50 \times 10^2 \text{ m}$

$$\theta = 45.0^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

Using the derivation shown in problem 1,

$$v_i = \sqrt{\frac{-a_y \Delta x}{2 (\sin \theta) (\cos \theta)}} = \sqrt{\frac{-(-9.81 \text{ m/s}^2)(9.50 \times 10^2 \text{ m})}{(2)(\sin 45.0^\circ)(\cos 45.0^\circ)}}$$

$$v_i = 96.5 \text{ m/s}$$

At the top of the arrow's flight:

$$v = v_x = v_i (\cos \theta) = (96.5 \text{ m/s})(\cos 45.0^\circ) = \boxed{68.2 \text{ m/s}}$$

3. $\Delta x = 27.5 \text{ m}$

$$\theta = 50.0^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

Using the derivation shown in problem 1,

$$v_i = \sqrt{\frac{-a_y \Delta x}{2 (\sin \theta) (\cos \theta)}} = \sqrt{\frac{-(-9.81 \text{ m/s}^2)(27.5 \text{ m})}{(2)(\sin 50.0^\circ)(\cos 50.0^\circ)}}$$

$$v_i = \boxed{16.6 \text{ m/s}}$$

4. $\Delta x = 44.0 \text{ m}$

$$\theta = 45.0^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

Using the derivation shown in problem 1,

$$\text{a. } v_i = \sqrt{\frac{-a_y \Delta x}{2 (\sin \theta) (\cos \theta)}} = \sqrt{\frac{-(-9.81 \text{ m/s}^2)(44.0 \text{ m})}{(2)(\sin 45.0^\circ)(\cos 45.0^\circ)}}$$

$$v_i = \boxed{20.8 \text{ m/s}}$$

- b.** At maximum height, $v_{y,f} = 0$ m/s

$$v_{y,f}^2 = v_{y,i}^2 + 2a_y \Delta y_{\max} = 0$$

$$y_{\max} = \frac{-v_{y,i}^2}{2a_y} = \frac{-v_i^2 (\sin \theta)^2}{2a_y} = \frac{-(20.8 \text{ m/s})^2 (\sin 45.0^\circ)^2}{(2)(-9.81 \text{ m/s}^2)} = 11.0 \text{ m}$$

The brick's maximum height is 11.0 m.

c. $y_{\max} = \frac{-v_{y,i}^2}{2a_y} = \frac{-(20.8 \text{ m/s})^2}{(2)(-9.81 \text{ m/s}^2)} = 22.1 \text{ m}$

The brick's maximum height is 22.1 m.

- 5.** $\Delta x = 76.5$ m

$$\theta = 12.0^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

At maximum height, $v_{y,f} = 0$ m/s.

$$v_{y,f}^2 = v_{y,i}^2 + 2a_y \Delta y_{\max} = 0$$

$$y_{\max} = \frac{-v_{y,i}^2}{2a_y} = \frac{-v_i^2 (\sin \theta)^2}{2a_y}$$

Using the derivation for v_i^2 from problem 1,

$$\Delta y_{\max} = \left[\frac{-a_y \Delta x}{2(\sin \theta)(\cos \theta)} \right] \frac{-(\sin \theta)^2}{2a_y} = \frac{\Delta x (\sin \theta)}{4(\cos \theta)} = \frac{\Delta x (\tan \theta)}{4}$$

$$\Delta y_{\max} = \frac{(76.5 \text{ m})(\tan 12.0^\circ)}{4} = 4.07 \text{ m}$$

- 6.** $v_{\text{runner}} = 5.82$ m/s

$$v_{i,\text{ball}} = 2v_{\text{runner}}$$

In x -direction,

$$v_{i,\text{ball}}(\cos \theta) = 2v_{\text{runner}}(\cos \theta) = v_{\text{runner}}$$

$$2(\cos \theta) = 1$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

- 7.** $v_i = 8.42$ m/s

$$\theta = 55.2^\circ$$

$$\Delta t = 1.40 \text{ s}$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

For first half of jump,

$$\Delta t_1 = \frac{1.40 \text{ s}}{2} = 0.700 \text{ s}$$

$$\Delta y = v_i (\sin \theta) \Delta t_1 + \frac{1}{2} a_y (\Delta t_1)^2 = (8.42 \text{ m/s})(\sin 55.2^\circ)(0.700 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(0.700 \text{ s})^2$$

$$\Delta y = 4.84 \text{ m} - 2.40 \text{ m} = 2.44 \text{ m}$$

The fence is 2.44 m high.

$$\Delta x = v_i (\cos \theta) \Delta t$$

$$\Delta x = (8.42 \text{ m/s})(\cos 55.2^\circ)(1.40 \text{ s}) = 6.73 \text{ m}$$

- 8.** $v_i = 2.2$ m/s

$$\theta = 21^\circ$$

$$\Delta t = 0.16 \text{ s}$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

$$\Delta x = v_i (\cos \theta) \Delta t = (2.2 \text{ m/s})(\cos 21^\circ)(0.16 \text{ s}) = 0.33 \text{ m}$$

Maximum height is reached in a time interval of $\frac{\Delta t}{2}$

$$\Delta y_{\max} = v_i (\sin \theta) \left(\frac{\Delta t}{2}\right) + \frac{1}{2} a_y \left(\frac{\Delta t}{2}\right)^2$$

$$\Delta y_{\max} = (2.2 \text{ m/s})(\sin 21^\circ) \left(\frac{0.16 \text{ s}}{2}\right) + \frac{1}{2}(-9.81 \text{ m/s}^2) \left(\frac{0.16 \text{ s}}{2}\right)^2$$

$$\Delta y_{\max} = 6.3 \times 10^{-2} \text{ m} - 3.1 \times 10^{-2} \text{ m} = 3.2 \times 10^{-2} \text{ m} = 3.2 \text{ cm}$$

The flea's maximum height is 3.2 cm.

Additional Practice F

Givens

1. $\mathbf{v}_{se} = 126 \text{ km/h north}$
 $\mathbf{v}_{gs} = 40.0 \text{ km/h east}$

Solutions

$$v_{ge} = \sqrt{v_{gs}^2 + v_{se}^2} = \sqrt{(40.0 \text{ km/h})^2 + (126 \text{ km/h})^2}$$

$$v_{ge} = \sqrt{1.60 \times 10^3 \text{ km}^2/\text{h}^2 + 1.59 \times 10^4 \text{ km}^2/\text{h}^2}$$

$$v_{ge} = \sqrt{1.75 \times 10^4 \text{ km}^2/\text{h}^2} = \boxed{132 \text{ km/h}}$$

$$\theta = \tan^{-1} \left(\frac{v_{se}}{v_{gs}} \right) = \tan^{-1} \left(\frac{126 \text{ km/h}}{40.0 \text{ km/h}} \right) = \boxed{72.4^\circ \text{ north of east}}$$

2. $\mathbf{v}_{we} = -3.00 \times 10^2 \text{ km/h}$
 $\mathbf{v}_{pw} = 4.50 \times 10^2 \text{ km/h}$
 $\Delta x = 250 \text{ km}$

$$v_{pe} = v_{pw} + v_{we} = 4.50 \times 10^2 \text{ km/h} - 3.00 \times 10^2 \text{ km/h} = 1.50 \times 10^2 \text{ km/h}$$

$$\Delta t = \frac{\Delta x}{v_{pe}} = \frac{250 \text{ km}}{1.50 \times 10^2 \text{ km/h}} = \boxed{1.7 \text{ h}}$$

3. $\mathbf{v}_{tw} = 9.0 \text{ m/s north}$
 $\mathbf{v}_{wb} = 3.0 \text{ m/s east}$
 $\Delta t = 1.0 \text{ min}$

$$\mathbf{v}_{tb} = \mathbf{v}_{tw} + \mathbf{v}_{wb}$$

$$v_{tb} = \sqrt{v_{tw}^2 + v_{wb}^2} = \sqrt{(9.0 \text{ m/s})^2 + (3.0 \text{ m/s})^2} = \sqrt{81 \text{ m}^2/\text{s}^2 + 9.0 \text{ m}^2/\text{s}^2}$$

$$v_{tb} = \sqrt{9.0 \times 10^1 \text{ m}^2/\text{s}^2}$$

$$v_{tb} = 9.5 \text{ m/s}$$

$$\Delta x = v_{tb} \Delta t = (9.5 \text{ m/s})(1.0 \text{ min}) \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = \boxed{570 \text{ m}}$$

$$\theta = \tan^{-1} \left(\frac{v_{wb}}{v_{tw}} \right) = \tan^{-1} \left(\frac{3.0 \text{ m/s}}{9.0 \text{ m/s}} \right) = \boxed{18^\circ \text{ east of north}}$$

4. $\mathbf{v}_{sw} = 40.0 \text{ km/h forward}$
 $\mathbf{v}_{fw} = 16.0 \text{ km/h forward}$
 $\Delta x = 60.0 \text{ m}$

$$\mathbf{v}_{sf} = \mathbf{v}_{sw} - \mathbf{v}_{fw} = 40.0 \text{ km/h} - 16.0 \text{ km/h} = 24.0 \text{ km/h toward fish}$$

$$\Delta t = \frac{\Delta x}{v_{sf}} = \frac{60.0 \text{ m}}{(24.0 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right)} = \boxed{9.00 \text{ s}}$$

5. $\mathbf{v}_{1E} = 90.0 \text{ km/h}$
 $\mathbf{v}_{2E} = -90.0 \text{ km/h}$
 $\Delta t = 40.0 \text{ s}$

$$\mathbf{v}_{12} = \mathbf{v}_{1E} - \mathbf{v}_{2E}$$

$$v_{12} = 90.0 \text{ km/h} - (-90.0 \text{ km/h}) = 1.80 \times 10^2 \text{ km/h}$$

$$\Delta x = v_{12} \Delta t = (1.80 \times 10^2 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right) (40.0 \text{ s}) = 2.00 \times 10^3 \text{ m} = 2.00 \text{ km}$$

The two geese are initially 2.00 km apart

6. $\mathbf{v}_{me} = 18.0 \text{ km/h forward}$
 $\mathbf{v}_{re} = 0.333 \mathbf{v}_{me}$
 $\quad = 6.00 \text{ km/h forward}$
 $\Delta x = 12.0 \text{ m}$

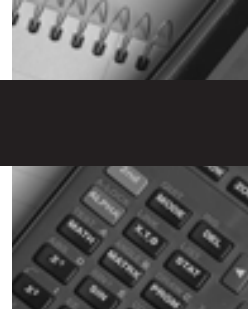
$$\mathbf{v}_{mr} = \mathbf{v}_{me} - \mathbf{v}_{re}$$

$$v_{mr} = 18.0 \text{ km/h} - 6.0 \text{ km/h} = 12.0 \text{ km/h}$$

$$\Delta t = \frac{\Delta x}{v_{mr}} = \frac{12.0 \text{ m}}{(12.0 \text{ km/h}) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) \left(\frac{1 \text{ km}}{10^3 \text{ m}} \right)}$$

$$\Delta t = \boxed{3.60 \text{ s}}$$

Two-Dimensional Motion and Vectors



Introduction to Vectors, p. 13

- {A, C, E, H, I}; {D, G}, {B, F, J}
- {A, D, H}, {B, C, G}, {I, J}
- {A, H}
- Both diagrams should show a vector **A** that is twice as long as the original vector **A**, but still pointing up. The first diagram should have the tip of $2\mathbf{A}$ next to the tail of **B**. The second diagram should have the tip of **B** next to the tail of $2\mathbf{A}$. The resultant vectors should have the same magnitude and direction, slanting towards the upper right.
- Both diagrams should show a vector **B** that is half as long as the original vector **B**. The first diagram should have the tip of **A** next to the tail of $-\mathbf{B}/2$, and $-\mathbf{B}/2$ should be pointing to the left. The second diagram should have the tip of $\mathbf{B}/2$ next to the tail of $-\mathbf{A}$, and $-\mathbf{A}$ should be pointing down. The resultant vectors should have the same magnitude but opposite directions. The first will slant towards the upper left. The second will slant towards the lower right.

Vector Operations, p. 14

- Check students' graph for accuracy. Shot 2: 110 m; 64 m Shot 4: 0 m; 14.89 m
- Shot 1: 45 m; 45 m Shot 3: 65 m; 33 m 3. 220 m

Projectile Motion, p. 15

- $\Delta t = v_i \sin \theta / g$
- $h = v_i^2 (\sin \theta)^2 / 2g$
- $x = v_i (\cos \theta) (\Delta t) = \frac{v_i^2 \sin \theta \cos \theta}{g}$
- $R = \frac{2v_i^2 \sin \theta \cos \theta}{g}$

5.

Launch angle	Maximum height (m)	Range (m)
15°	8.5	130
30°	32	220
45°	64	250
60°	96	220
75°	119	130

Relative Motion, p. 16

- $\mathbf{v}_{BL} = \mathbf{v}_{BW} + \mathbf{v}_{WL}$
- Student diagrams should show \mathbf{v}_{BW} twice as long as \mathbf{v}_{WL} but both are in the same direction as \mathbf{v}_{BL} , which is long as both together.
- Student diagrams should show \mathbf{v}_{WL} and \mathbf{v}_{BW} , longer and opposite in direction. The vector \mathbf{v}_{BL} should be as long as the difference between the two, and in the same direction and in the same direction as \mathbf{v}_{BW} .
- Student diagrams should show \mathbf{v}_{WL} and \mathbf{v}_{BW} at a right angle with \mathbf{v}_{BL} forming the hypotenuse of a right triangle.
- 6.0 km/h, due east
 - 2.0 km/h, due west
 - 4.5 km/h, $\theta = 26.6^\circ$

Mixed Review, pp. 17–18

1. a. The diagram should indicate the relative distances and directions for each segment of the path.

b. 5.0 km, slightly north of northwest

c. 11.0 km

2. a. The same

b. Twice as large

c. 1.58

3. a. 2.5 m/s, in the direction of the sidewalk's motion

b. 1.0 m/s, in the direction of the sidewalk's motion

c. 4.5 m/s, in the direction of the sidewalk's motion

d. 2.5 m/s, in the direction opposite to the sidewalk's motion

e. 4.7 m/s, $\theta = 32^\circ$

4. a. 4.0×10^1 seconds

b. 6.0×10^1 seconds