## Chapter 3

## Section 2 Vector Operations

## Preview

- Objectives
- Coordinate Systems in Two Dimensions
- Determining Resultant Magnitude and Direction
- Sample Problem
- Resolving Vectors into Components
- Adding Vectors That Are Not Perpendicular


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## Objectives ,

- Identify appropriate coordinate systems for solving problems with vectors.
- Apply the Pythagorean theorem and tangent function to calculate the magnitude and direction of a resultant vector. -
- Resolve vectors into components using the sine and cosine functions. v
- Add vectors that are not perpendicular.


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## Coordinate Systems in Two Dimensions ,

- One method for diagraming the motion of an object employs vectors and the use of the $x$ and $y$-axes.
- Axes are often designated using fixed directions. v
- In the figure shown here, the
 positive $y$-axis points north and the positive $x$-axis points east.


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## Determining Resultant Magnitude and Direction

- In Section 1, the magnitude and direction of a resultant were found graphically. v
- With this approach, the accuracy of the answer depends on how carefully the diagram is drawn and measured.
- A simpler method uses the Pythagorean theorem and the tangent function.


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## Determining Resultant Magnitude and Direction, continued

## The Pythagorean Theorem

- Use the Pythagorean theorem to find the magnitude of the resultant vector. v
- The Pythagorean theorem states that for any right triangle, the square of the hypotenuse-the side opposite the right angle-equals the sum of the squares of the other two sides, or legs.

$$
\begin{gathered}
c^{2}=a^{2}+b^{2} \\
(\text { hypotenuse })^{2}=(\operatorname{leg} 1)^{2}+(\operatorname{leg} 2)^{2}
\end{gathered}
$$



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## Determining Resultant Magnitude and Direction, continued

## The Tangent Function

- Use the tangent function to find the direction of the resultant vector. .
- For any right triangle, the tangent of an angle is defined as the ratio of the opposite and adjacent legs with respect to a specified acute angle of a right triangle.

$$
\text { tangent of angle } \theta=\frac{\text { opposite leg }}{\text { adjacent leg }}
$$



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## Sample Problem ,

Finding Resultant Magnitude and Direction , An archaeologist climbs the Great Pyramid in Giza, Egypt. The pyramid's height is 136 m and its width is $2.30 \times 10^{2} \mathrm{~m}$. What is the magnitude and the direction of the displacement of the archaeologist after she has climbed from the bottom of the pyramid to the top?

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## Sample Problem, continued

1. Define ${ }_{v}$

Given:

$$
\begin{aligned}
& \Delta y=136 \mathrm{~m} \\
& \Delta x=1 / 2(\text { width })=115 \mathrm{~m}
\end{aligned}
$$

Unknown:

$$
d=? \quad \theta=?
$$



Diagram:
Choose the archaeologist's starting position as the origin of the coordinate system, as shown above.

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## Sample Problem, continued

## 2. Plan ,

Choose an equation or situation: The
Pythagorean theorem can be used to find the magnitude of the archaeologist's displacement.
The direction of the displacement can be found by using the inverse tangent function.

$$
d^{2}=\Delta x^{2}+\Delta y^{2} \quad \tan \theta=\frac{\Delta y}{\Delta x}
$$

Rearrange the equations to isolate the unknowns:

$$
d=\sqrt{\Delta x^{2}+\Delta y^{2}} \quad \theta=\tan ^{-1}\left(\frac{\Delta y}{\Delta x}\right)
$$

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## Sample Problem, continued

3. Calculate

$$
\begin{array}{ll}
d=\sqrt{\Delta x^{2}+\Delta y^{2}} & \theta=\tan ^{-1}\left(\frac{\Delta y}{\Delta x}\right) \\
d=\sqrt{(115 \mathrm{~m})^{2}+(136 \mathrm{~m})^{2}} & \theta=\tan ^{-1}\left(\frac{136 \mathrm{~m}}{115}\right) \\
d=178 \mathrm{~m} & \theta=49.8^{\circ}
\end{array}
$$

4. Evaluate

Because $d$ is the hypotenuse, the archaeologist's displacement should be less than the sum of the height and half of the width. The angle is expected to be more than $45^{\circ}$ because the height is greater than half of the width.

